JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTURIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

1st YEAR 1st SEMESTER 2023/2024 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 803

COURSE TITLE: FUNCTIONAL ANAYSIS I

EXAM VENUE: **STREAM: (Msc. Pure Mathematics)**

DATE: **EXAM SESSION:**

TIME: 3.00HRS

Instructions:

1. **Answer any THREE questions only**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

QUESTION ONE [20 MARKS]

a) Define a normed space and provide examples. [5marks]

b) Prove that every finite-dimensional normed space is a Banach space. Discuss the implications of this result in the study of normed spaces. [15marks]

QUESTION TWO [20 MARKS]

a) Discuss Hilbert space properties. [6marks]

b) Given a bounded linear operator on a Hilbert space, illustrate how you would determine its spectral properties, including its spectrum and eigenvalues. Provide examples to illustrate your answer. [14marks]

QUESTION THREE [20 MARKS]

a) Give examples of normed spaces that are not Banach spaces and explain why they fail to satisfy the completeness property [8marks]

b) Highlight the role of the Hahn-Banach theorem in establishing duality theorems. [12marks]

QUESTION FOUR [20 MARKS]

a) Prove the Banach Fixed-Point Theorem and explore its significance in the study of Banach spaces. [10marks]

b) Discuss the consequences of the Open Mapping Theorem in the study of bounded linear operators.

[10marks]

QUESTION FIVE [20 MARKS]

a) Let H be a Hilbert space. Show that any bounded linear operator T: $H \rightarrow H$ has a unique adjoint operator T*. Discuss properties of the adjoint operator and its relationship with the original operator. [8marks]

b) Explore the concept of weak convergence in Banach spaces. Provide examples illustrating the difference between weak and norm convergence, and explore the relationship between weak convergence and weak* convergence. [12marks]