JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

1^{ST} YEAR 1^{ST} SEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 805

COURSE TITLE: General Topology I

EXAM VENUE: **STREAM: (MSc. PURE MATHEMATICS)**

DATE: **EXAM SESSION: 2.00 – 5.00 PM**

TIME: 3.00 HOURS

Instructions:

1. **Answer any THREE questions only.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE 20 MARKS

a) If $X = \{a, b, c\}$ and τ is a topology on X, with $\{a\} \in \tau, \{b\} \in \tau$ and $\{c\} \in \tau$. Prove that τ is a discrete topology. (7 marks)

b) Let (X, τ) be any topological space. Verify that the intersection of any finite number of members of τ is a member inτ. (7 marks)

c) List all possible topologies of the set $Y = \{a, b, c\}$. (6 marks)

QUESTION TWO (20 MARKS)

a) Let B be a basis for a topology τ on a non-empty set X If B_1 is a collection of subsets of X such that $\tau \supseteq B_1 \supseteq B$. Prove that B_1 is also a basis for τ. (8 marks)

b) Let $C[0,1]$ be the set of all continuous real-valued functions on $[0,1]$. Show that the collection M where $M = \{m(f,\epsilon): f \in C[0,1] \text{ and } \epsilon \text{ is a positive number}\}$ and $m(g,\epsilon) = \{g: g \in C[0,1] \text{ and } \int_0^1 |f-g| < \epsilon\}$ is a basis for a topology τ on $C[0,1]$. (12 marks)

QUESTION THREE (20 MARKS)

a) Let (X,τ) be any topological space. Prove that any arbitrary intersection of closed subsets of X are closed and finite union of closed subsets of X are closed. (8 marks)

b) Let $X = \{a,b,c,d,e\}$ and $\tau = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,e\}\}$. List the members of the induced topologies τ_Y on $Y = \{a,c,e\}$ and τ_Z on $Z = \{b,c,d,e\}$. (4 marks)

c) Show that a topological space (X,τ) is a τ_1-space if and only if every subset consist one point is closed. (8 marks)

QUESTION FOUR (20 MARKS)

a) State and prove the following : (6 marks)
i) Urysohns lemma
ii) Tietze's extension theorem.

b) Show that if A is closed subspace of Lindelof space (X,τ) then (A,τ_A) is also lindelof. (6 marks)

c) Let (X,τ) be a topological space and R be an equivalent relation on X. Suppose $X|R$ has idendification topological τ^*. Prove that the identification map $f: (X,\tau) \to (X|R,\ \tau^*)$ is continuous. (8 marks)

QUESTION FIVE (20 MARKS)

a) Show that any two open intervals in $\mathbb{R}$ are homeomorphic. (7 marks)

b) Show that any 2^{nd} countable space is first countable. (6 marks)

c) Let (X,τ_X) and (Y,τ_Y) be two topological spaces. Show that $f: X \to Y$ is continuous if and only if $f^{-1}(B) \in \tau$ for all $B \in \mathcal{B}$ where $\mathcal{B}$ is a basis for τ_Y. (7 marks)