JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR THE AWARD OF THE DEGREE OF MASTER IN PURE MATHEMATICS

1ST YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 820

COURSE TITLE: OPERATOR THEORY I

EXAM VENUE: **STREAM: MSC. PURE MATHEMATICS**

DATE: **EXAM SESSION:**

TIME: 3.00 HOURS

Instructions:

1. **Answer any THREE questions only.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)

a) Define the operator norm of a linear operator T and explain how it is computed. Provide an example of calculating the operator norm for a specific operator. (10marks)

b) i) Find the eigenvalues and eigenvectors of the matrix operator: $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$(3marks)

ii) Analyze the spectrum of the operator A from (i). Is it compact? Normal? Justify your answer. (7marks)

QUESTION TWO (20 MARKS)

a) Explain the concept of the dual space and its properties and briefly discuss the relationship between the dual space and the space itself. (10marks)

b) Describe how operator theory plays a role in quantum mechanics. Provide a specific example of a quantum mechanical system and its operator-based representation. (10marks)

QUESTION THREE (20 MARKS)

a) State Gelfand's formula and explain its connection to the spectral radius. (8marks)

b) Describe how the spectral radius relates to the stability of dynamical systems governed by linear operators. (12marks)

QUESTION FOUR (20 MARKS)

a) i) Show that the operator T:ℓ^2(ℝ)→ ℓ^2 (ℝ) defined by $T(x)$=(x_1, x_2, x_3,…) is bounded. (5marks)

ii) Explain how the compactness of an operator affects its spectrum. (5marks)

b) State Banach-Steinhaus theorem and explain its significance in Operator Theory.

(10marks)

QUESTION FIVE (20 MARKS)

Discuss the importance of numerical ranges in functional analysis. Explain how they can be used to analyze the properties of linear operators, including spectral properties, invertibility, and compactness. Include relevant examples and theorems in your discussion. (20marks)