JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

1ST YEAR 1ST SEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: 822

COURSE TITLE: BANACH ALGEBRA I

EXAM VENUE: **STREAM: (Msc. Pure Mathematics)**

DATE: **EXAM SESSION: TWO**

TIME: 3.00 HOURS

Instructions:

1. **Answer any THREE questions only**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE [20 MARKS]

a) Analytically describe: A normed algebra; sub-algebra and Banach algebra. (6 marks)

b) Prove that the union of two Banach algebras is Banach algebra. (10 marks)

c) State and prove the condition under which $l^{\infty}(S)$ the set of all bounded valued on nonempty set S is a unital Banach algebra. (4 marks)

QUESTION TWO [20 MARKS]

a) Prove that if Z is the set off all intergers with counting measure then $l^1(Z)$ is Banach algebras. (10 marks)

b) Prove that any nonempty open subset of irreducible Banach algebra is dense and irreducible. Moreover, prove that if Y is a subset of a Banach algebra X, which is irreduciblein its induced sub-algebra then the closure of Y is also irreducible. (10 marks)

QUESTION THREE [20 MARKS]

a) Describe: Trivial ideal, modula ideal, Maximal ideal and prime ideal. (4 marks)

b) Let $C_o(\Omega)$ be an algebra and $M_w = \{f \in C_o(\Omega) : f(w) = 0\}$. Show that M_w is a modula ideal. (8 marks)

c) Describe the process of unitization of normed algebras. (8 marks)

QUESTION FOUR [20 MARKS]

a) Differentiate between the left inverse and right inverse in Banach algebras. (4 marks)

b) Show that an inverse in a Banach algebra is unique. (6 marks)

c) Let Ω be an algebra and e its unity. Prove that if $x \in \Omega$ and $\|ex\| \prec 1$ then there exist x^{-1} such that $\|x^{-1}\| \leq \dfrac{e}{e\|ex\|}$. (10 marks)

QUESTION FIVE [20 MARKS]

a) Define multiplicative linear functional ϕ and show that $\|\phi\| = 1$. (6 marks)

b) Define a C^* algebra and show that a linear functional f is bounded with $\| f \| = f(e)$. (10 marks)

c) Describe a Gelfand transform and hence show that it is a contractive Banach algebraic homomorphism. (4 marks)