JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTURIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF ……..

1ST YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: WMB 9108

COURSE TITLE: CALCULUS I

EXAM VENUE: **STREAM:**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

a) Explain why the limit below do not exist:

$\lim_{x\to 0}\frac{x}{|x|}$ (3 marks)

b) Find $\lim_{x\to+\infty}\frac{2x^2+1}{x^2+6x-4}$. Hence, give geometrical interpretation of your answer. (4marks)

c) Determine the point of discontinuity (if any) of the function $f(x)$

$f(x)=\frac{2x^2-5x-3}{x-3}$.

If the discontinuity is removable, define the function to make it continuous. (4 marks)

d) Using the definition, find the derivative $f'(x)$ of the function $f(x)=x^2+2x-1$ (4 marks)

e) Find the critical numbers of $f(x)=x^3-3x+1$ and determine whether they yield relative maximum, relative minimum or inflection points (5 marks)

f) Find the derivative of the function

$y=\left(\frac{x-1}{3x^2-2}\right)^{-1}$ (5 marks)

g) Given that $f(x)=\sin x$, prove that $f'(x)=\cos x$ (5 marks)

QUESTION TWO (20 marks)

a) Show that

$\frac{d}{dx}(uv)=u\frac{dv}{dx}+\frac{du}{dx}v$ (6 marks)

b) Given that $f(x)=\frac{x\ln x}{1+\ln x}$, find $f'(x)$ (4 marks)

c) Find the derivative of $y=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$ (6 marks)

d) Find the derivative of the function

$f(x)=\frac{\sec x}{1+\tan x}$ (4 marks)

QUESTION THREE (20 marks)

a) If $\frac{x-y}{2x+2y}=2x$, find $\left.\frac{dy}{dx}\right|_{(2,1)}$ by implicit differentiation. (6 marks)

b) Find the second order derivative of the function:

$g(t)=\frac{4}{(t+2)^2}$ (5 marks)

c) If $x=a\cos\theta$, $y=b\sin\theta$, show that

$\frac{d^2y}{dx^2}=-\frac{b}{a^2}\cos ec^3\theta$ (9 marks)

QUESTION FOUR (20 marks)

i. The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 4.5t^2 - 12t + 36$, where x is expressed in feet and t in seconds. Determine:

   a) the time at which the velocity will be zero, (3 marks)
   b) the position and distance traveled by the particle at that time, (4 marks)
   c) the acceleration of the particle at that time, (3 marks)
   d) the distance traveled by the particle from $t = 3s$ to $t = 5s$. (4 marks)

ii. The demand function for the Luminar desk lamp is given by

$$P = f(x) = -0.1x^2 - 0.4x + 35$$

where x is the quantity demanded (measured in thousands) and p is the unit price in dollars. What is the rate of change of the unit price when the quantity demanded is 10,000 units (x=10)? What is the unit price at that level of demand? (6 marks)

QUESTION FIVE (20 marks)

i. Use Logarithmic differentiation to find the derivative of y with respect to x

$$y = \frac{x(x-1)^{3/2}}{\sqrt{x+1}}$$ (7 marks)

ii. Find the derivative of y with respect to x, given that

$$\ln xy = e^{x+y}$$ (6 marks)

iii. Find the derivative of y with respect to x, where

$$y = \ln \sqrt[3]{\frac{x-1}{x+1}}$$ (7 marks)

The demand function for the Luminar desk lamp is given by

$$P = f(x) = -0.1x^2 - 0.4x + 35$$

where x is the quantitydemanded (measured in thousands) and p is the unit price in dollars.

a) Find $f'(x)$.
b) What is the rate of change of the unit price when the quantitydemanded is 10,000 units (x=10)? What is the unit price at that level of demand?