JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCE
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION/ ACTUARIAL SCIENCES

2ND YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

(MAIN)

COURSE CODE: WMB 9208

COURSE TITLE: INTRODUCTION TO ANALYSIS

EXAM VENUE: **STREAM: (BSc. Actuarial/Education)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

a) Define the following terms

i) An open set. (2 marks)

ii) A point of accumulation. (2 marks)

b) Show that intersection of any two open sets is open. (3 marks)

c) Show that there is no number whose square root is 2. (6 marks)

d) If $A = \{2, 3,6,8\}$, $B = \{1, 2.4.6.9\}$ and $U = \{1,2, \dots ,9\}$. Find

i) $A^C \cap B$ (2 marks)

ii) $A \setminus B$. (2 mark)

e) Given that $A = \{2,8\}$, $B = \{x, y, z\}$

i) Find $B \times A$. (2 marks)

ii) Determine the power set of A. (3 marks)

f) State the point of discontinuity Determine the limit of the following function and.

$f(x) = \frac{x^2-x-12}{x-4}$ as $x \to 4$. (5 marks)

g) Determine the boundary of the set $D = \{(x, y): x^2 - y^2 < 1\}$ at the point $x = \frac{1}{2}$. (3 marks)

QUESTION TWO (20 marks)

a) Let $f(x) = 2x^2$, $g(x) = 2x - 1$ and $h(x) = \frac{3}{4}x - 2$

i) Show that $f \circ g \neq g \circ f$. (4 marks)

ii) Find $f \circ g(1)$. (2 marks)

iii) Determine $h^{-1}(x)$, the inverse of h. (3 marks)

b) Use the definition of limits to show that the limit of $f(x) = 5x - 3$ is equal to 7 as $x \to 2$. (4 marks)

c) Let $g(x) = 3x - 4$. Prove that g is uniformly continuous on $\mathbb{R}$. (4 marks)

d) Let $X = \{2,5\}$. Determine a relation, R on X such that $R = \{(x, y): x \leq y\}$. Hence or otherwise obtain R^{-1}. (3 marks)

QUESTION THREE (20 marks)

a) Define the terms
   i) A Monotonically decreasing sequence. (2 marks)

   ii) A convergent sequence. (2 marks)

b) Show that if the limit of a convergent sequence exists, then it is unique. (5 marks)

c) Given the sequence $x_n = 1 - \frac{1}{n}, n \in \mathbb{N}$. List the first 3 elements of the sequence and determine its monotonicity. (3 marks)

d) Determine the limit of the function $f(x) = \frac{2x^2+5x-4}{4x^2-2x}$, as $x \to \infty$. (3 marks)

e) Show that the upper half-plane $Q = \{(x, y): y > 0\}$ is open in $\mathbb{R}^2$. (5 marks)

QUESTION FOUR (20 marks)

a) Define the following terms
   i) Lower bound of a set. (2 marks)
   ii) Supremum of a set. (2 marks)

b) Determine the lower bound and upper bound of the following sets.
   i) $P = \{-1 \leq p \leq 3\}$. (2 marks)
   ii) $Q = \left\{1 - \frac{1}{n}\right\}, n \in \mathbb{N}$. (2 marks)

c) Let X be a bounded set. If the supremum does exist then show that the supremum is unique. (4 marks)

d) Show that the union of any two open sets is open. (3 marks)

e) Show that a set is closed if and only if its complement is open. (5 marks)

QUESTION FIVE (20 marks)

a) By using the definition of a field, determine whether the set of integers $\mathbb{Z}$, is a field or not. (10 marks)

b) State and prove the Bolzano-Weierstrass Theorem of sequences. (10 marks)