JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

3RD YEAR 1ST SEMESTER 2016/2017 ACADEMIC YEAR

MAIN

REGULAR

COURSE CODE: SPH 303

COURSE TITLE: QUANTUM MECHANICS I

EXAM VENUE: **STREAM: EDUCATION**

DATE: **EXAM SESSION:**

TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful constants

$\hbar = 1.054\ x\ 10^{-34}\ Js$

$1eV = 1.6\ x\ 10^{-19}\ J$

$h = 6.63\ x\ 10^{-34} Js$

$Mass\ of\ electron = 9.1\ x\ 10^{-31} kg$

$Mass\ of\ proton = 1.672\ x\ 10^{-27}\ kg$

SECTION A

QUESTION 1(30 MARKS)

a)

i. Calculate the **de Broglie** wavelength for an electron having kinetic energy of **1** $\boldsymbol{eV}$.

(3 marks)

ii. In the **double-slit experiment**, two waves defined by $\boldsymbol{\psi_1 = \frac{1}{\sqrt{2}} e^{ix}}$ and $\boldsymbol{\psi_2 = e^{ix}}$ pass through the slits. Determine the probability density on the screen.

(4 marks)

b) Explain the **probabilistic interpretation** of quantum mechanics.

(2 marks)

c) Derive the **time-independent** Schrödinger equation. **(4 marks)**

d) Define the following terms as used in quantum mechanics.

i. Bound state. **(1 mark)**

ii. Tunnelling. **(1 mark)**

e) The **expectation value** of the position of a particle described by the wave function $\boldsymbol{\psi = \frac{1}{2} x}$ limited to the x-axis between $\boldsymbol{x = 0}$ and $\boldsymbol{x = b}$ is $\frac{1}{16}$. Find the value of $\boldsymbol{b}$.

(3 marks)

f) Prove that the **commutation brackets**

$\left[\hat{A}, \left[\hat{B}, \hat{C}\right]\right] + \left[\hat{B}, \left[\hat{C}, \hat{A}\right]\right] + \left[\hat{C}, \left[\hat{A}, \hat{B}\right]\right] = 0$ **(4 marks)**

g) An **eigenfunction** of the operator $\frac{d^2}{dx^2}$ is $\boldsymbol{\psi = e^{2x}}$. Find the corresponding **eigenvalue**.

(3 marks)

h) An operator is defined by $\hat{D}_x = \frac{\partial}{\partial x}$. Determine the **Heisenberg's uncertainty product** in the measurement of $\hat{x}$ and $\hat{D}_x$. **(4 marks)**

i) State **ONE** postulate of Quantum mechanics. **(1 mark)**

SECTION B

Answer any TWO questions in this section.

QUESTION 2 (20 MARKS)

a) Solve the one –dimensional time-independent Schrödinger equation for a particle in a **step potential with a step height** greater than the total energy.

(15 marks)

b) Estimate the **penetration distance** for a very small dust particle of mass $\boldsymbol{5x10^{-14}\ kg}$ moving at a velocity of $\boldsymbol{0.02\ ms^{-1}}$ if the particle impinges on a potential step of height twice its kinetic energy. **(5 marks)**

QUESTION 3 (20 MARKS)

a) Show that the general **wave function for a free particle** in one-dimensional motion is given by $\boldsymbol{\psi(x) = a\cos\frac{1}{\hbar}(px + \hbar\theta)}$ where the symbols have their usual meanings.

(16 marks)

b) Show that the constant $\boldsymbol{E}$ in the time-independent Schrödinger equation is the **expectation** value of the Hamiltonian. **(4 marks)**

QUESTION 4 (20 MARKS)

a) A measurement establishes the position of a proton with an accuracy of $\boldsymbol{1.00\ x\ 10^{-11}\ m}$. Use **Heisenberg's uncertainty principle** to find the uncertainty in the proton's position $\boldsymbol{1.00\ s}$ later. **(9 marks)**

b) The Hamiltonian of a quantized **linear harmonic oscillator** is given by

$\boldsymbol{\hat{H} = \hbar\omega\left(\hat{a}^{+}a + \frac{1}{2}\right)}$**.** Obtain an expression for the energy at second energy level.

(7 marks)

c) Calculate the frequency of a **harmonic oscillator** at ground state if the ground state energy is $\boldsymbol{3.4\ eV}$. **(4 marks)**

QUESTION 5 (20 MARKS)

a) Consider a particle in **an infinite potential** box (potential zero inside the box and infinity outside) which extends from $\boldsymbol{x = 0}$ to $\boldsymbol{x = \pi}$ Å. Obtain an expression for the energy eigenvalues in terms of $\boldsymbol{n}$ where $\boldsymbol{n = 1, 2}, ...$

(9 marks)

b) Suppose an electron has a wave function $\boldsymbol{\psi(x) = cx^3e^{-\alpha|x|}}$ where $\boldsymbol{\alpha}$ is a constant.

i. Find the constant $\boldsymbol{c}$ that ensures the given wave function is properly **normalized**. You may use the standard integral

$$\int_0^\infty x^n e^{-bx} dx = \frac{n!}{b^{n+1}}$$ **(6 marks)**

ii. Find the **expectation** value of $\boldsymbol{x}$. **(5 marks)**