JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION

$2^{ND}$ YEAR $2^{ND}$ SEMESTER 2016/2017

# MAIN REGULAR

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**COURSE CODE: SPH 203**

**COURSE TITLE: MATHEMATICAL METHODS FOR PHYSICS I**

**EXAM VENUE: LAB 2** **STREAM: (B.Ed Sc)**

**DATE: 09/04/2017** **EXAM SESSION: 8.00 A.M. – 10.00 A.M.**

**TIME: 2:00 HRS**

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**INSTRUCTIONS:**

1. **Attempt question 1 (compulsory) and ANY other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**Question 1**

**(a) (i)** Compute the limit

$$\lim_{x \to 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$$

**(ii)** Show that $\cosh^2 x - \sinh^2 x = 1$

(5 Marks)

**(b)** Find the derivative of the function

$$y(x) = \sin(3x^2 + 2)^2$$

(5 Marks)

**(c)** Evaluate the following integral,

$$\int x \sin 5x \, \mathrm{d}x$$

(5 Marks)

**(d) (i)** List down any four axioms of a vector space.
**(ii)** Investigate whether the following is a basis for $\mathbb{R}^2$,

$$\left\langle \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\rangle$$

(6 Marks)

**(e)** Determine a scalar $c$ such that the following vectors are orthogonal,

$$\vec{a} = 2\hat{\mathbf{\imath}} - c\hat{\mathbf{\jmath}} + 3\hat{\mathbf{k}}, \qquad \vec{b} = 3\hat{\mathbf{\imath}} + 2\hat{\mathbf{\jmath}} + 4\hat{\mathbf{k}}$$

(4 Marks)

**(f)** The potential energy between two atoms in a diatomic molecule is given by $U(x) = 2/x^{12} - 1/x^6$. Find the minimum potential energy between the two atoms.

(5 Marks)

**Question 2**

**(a)** Use de L'Hôpital's rule to evaluate,

$$\lim_{x \to \infty} \left[ \sqrt{x^2 + x} - x \right]$$

(4 Marks)

**(b)** A series is given by

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Find the radius and interval of convergence for the series.

(6 Marks)

**(c)** Given the sine series,

$$\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

use calculus to obtain the cosine series. Hence calculate the cosine of 0.1 radians.

(5 Marks)

**(d)** Evaluate the following geometric sum,

$$\frac{1}{4} + \frac{1}{12} + \frac{1}{36} + \frac{1}{108} + \cdots + \frac{1}{2916}$$

(5 Marks)

## Question 3

Use an appropriate method to evaluate,

$$\int_0^{\pi/2} \frac{\mathrm{d}x}{3 + 2\cos x + 3\sin x}$$

(20 Marks)

## Question 4

**(a)** A curve $C$ has the parametric equations $x = 2t - 5$, $y = t^2 - 4t + 3$. Find an equation of the tangent line to $C$ that is parallel to the line $y = 3x + 1$.

(6 Marks)

**(b)** A boy wishes to cut a 2.5 m long piece of wire into two pieces. One piece will be bent into a circular shape and the other into the shape of a square. Find the length that each wire should have so that the sum of the areas is a maximum (you may leave your answers in terms of $\pi$).

(10 Marks)

**(c)** Differentiate with respect to $x$:

**(i)** $y = x^2 e^{3x}$

**(ii)** $y = \ln(2x^3 + x^2)$

(4 Marks)

**Question 5**

**(a)** **(i)** Given that $\vec{a} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\vec{b} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, evaluate the vector product $\vec{a} \times \vec{b}$.
**(ii)** Obtain the component of $\vec{a}$ in the direction of $\vec{b}$.

(5 Marks)

**(b)** The coordinates $(x, y, z)$ of points A and B on a cartesian coordinate system are $(1, 1, 1)$ and $(3, 5, 6)$ respectively.

**(i)** Write down the position vectors $\vec{a}$ and $\vec{b}$ of the points A and B, hence find

**(ii)** the displacement vector $\overrightarrow{AB}$.

**(iii)** the unit vector parallel to $\overrightarrow{AB}$.

**(iv)** the coordinates of a point P on AB such that the ratio of the displacement vectors $\overrightarrow{AB}$ and $\overrightarrow{PB}$ is $\overrightarrow{AB} : \overrightarrow{PB} = 1 : 3$.

(12 Marks)

**(c)** Consider a particle whose position vector is defined as,

$$\vec{r}(t) = (2t^2 - 1)\hat{\mathbf{i}} + (4t^3 - 3)\hat{\mathbf{j}}$$

Find an expression for the acceleration at any time $t$.

(3 Marks)