JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF HEALTH SCIENCES

UNIVERSITY EXAMINATION FOR MSc IN APPLIED MATHEMATICS

1st YEAR 1st SEMESTER 2016/2017 ACADEMIC YEAR

KISUMU CAMPUS

COURSE CODE: SMA840
COURSE TITLE: METHODS OF APPLIED MATHEMATICS I

EXAM VENUE: **STREAM: MSc Y1S1**

TIME: 3 HOURS **EXAM SESSION:**

Instructions:

1. **Answer any THREE questions.**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

Question1 [20 marks]

(a). Compute the Laplace transform of (i) $te^{-140t}\sin 13t$ (ii) $\left[\dfrac{\cos 3t-\cos 2t}{t}\right]$ [10 marks]

(b) If $F(s)=\dfrac{4s+13}{(s^2+16)(s+2)}$ use the Laplace transforms to find $f(t)$. [10 marks]

Question 2[20 marks]

As an electric locomotive travels down a track at the speed V ,
the pantograph electrical system pushes up the line with a force P which is known to satisfy the hyperbolic partial differential equation

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}+\frac{P}{\rho V}\delta\left(t-\frac{x}{V}\right)\ ;\ 0<x<L,\ t>0\,,$$

subject to $u(0,t)=u(L,t)=0$ for $t>0$

and $u(x,0)=u_t(x,0)=0$ where $,t=0$.

Use Laplace transform to determine $u(x,t)$, the behavior of the overhead wire as pantograph passes between two supports of the electrical cable that are located a distance L if initially the wires are at rest. [20marks]

Question3 [20marks]

Define a system of first order linear ordinary differential equations by

$$\dot{\underset{\sim}{X}}=\begin{pmatrix}1 & 1 & 1\\ 2 & 1 & -1\\ -8 & -5 & -3\end{pmatrix}\underset{\sim}{X}\quad .$$

(a) Compute e^{At} the exponential matrix $A=\begin{pmatrix}1 & 1 & 1\\ 2 & 1 & -1\\ -8 & -5 & -3\end{pmatrix}$ [10 marks]

(b)Verify that $\left[e^{At}\right]_{t=0}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ [6 marks]

(c)Show that $e^{At}=\Phi(t)\Phi^{-1}(0)$ where $\Phi(t)$ is the fundamental matrix of the system.

[4 marks]

Question4 [20marks]

(a) Apply the Laplace transform method to solve the simultaneous equations

$\frac{dx}{dt}-5y=-3x,\ \frac{dy}{dt}-y=-x:\ x(0)=2, y(0)=1$ [16 marks]

(b) Determine the value of y which satisfies the ordinary differential equation

$xy''+(1-x)y'+my=0;$ subject to $y(0)=\alpha,\ y'(0)=\beta$, m positive integer.

[4 marks]

Question5 [20marks]

Solve the initial boundary value problem

$u_t=u_{xx}$, $0<x<1$, $t>0$ satisfying the conditions

$u(0,t)=0,\ u(1,t)=1\quad 0<x<1,\ t>0$

$u(x,0)=1+\sin\pi x+\cos\pi x,\quad 0<x<1$ [20marks].

LAPLACE TRANSFORMS TABLE

$f(t)$	Laplace transform of $f(t)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
e^{at}	$\frac{1}{s-a}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{(s+a)}{(s+a)^2+b^2}$
$e^{-at}t^n$	$\frac{\Gamma(n+1)}{(s+a)^{n+1}} \quad n>-1$
t^n	$\frac{n!}{s^{n+1}}$
$e^{-at}t^n$	$\frac{n!}{(s+a)^{n+1}}$
$\frac{dy}{dt}$	$sY-y_0 \quad ; \quad Y=L(y)$
$\frac{d^2y}{dt^2}$	$s^2Y-sy_0-y_0' \;\; ; \;\; Y=L(y)$
$\frac{d^ny}{dt^n}$	$s^nY-s^{n-1}y_0-s^{n-2}y_0-s^{n-3}y_0-....$ $-sy^{(n-2)}(0)-y^{(n-1)}(0)$
u_t	$sU(x,s)-u(x,0); \quad U(x,s)=L[u(x,t)]$
u_{tt}	$s^2U(x,s)-su(x,0)-u_t(x,0);$
u_{x^m}	$\frac{d^m}{dx^m}(U(x,s))$
u_{xt}	$s\frac{d}{dx}U(x,s)-\frac{d}{dx}u(x,0)$
$J_0(t)$	$\frac{1}{\sqrt{s^2+1}}$

$t^n f(t)$ $\frac{f(t)}{t}$	$(-1)^n \frac{d^n\{F(s)\}}{ds^n}$ $\int_s^{\infty} F(s)\,ds$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f(0) - ....sf^{(n-2)}(0) - f^{(n-1)}(0)$
$J_0(t)$	$\frac{1}{\sqrt{1+s^2}}$
$\frac{\partial u(x,t)}{\partial t}$	$sU(x,s) - u(x,0)$
$\frac{\partial^2 u(x,t)}{\partial t^2}$ e^{At}	$s^2 U(x,s) - su(x,0) - u_t(x,0)$ $(sI - A)^{-1}$

LAPLACE TRANSFORMS

$\frac{\partial u(x,t)}{\partial t}$	$sU(x,s) - u(x,0)$
$\frac{\partial^2 u(x,t)}{\partial t^2}$	$s^2 U(x,s) - su(x,0) - u_t(x,0)$
$\frac{\partial u(x,t)}{\partial x}$	$\frac{dU(x,s)}{dx}$
$\frac{\partial^2 u(x,t)}{\partial x^2}$	$\frac{d^2 U(x,s)}{dx^2}$
$\frac{\partial^2 u(x,t)}{\partial t \partial x}$	$s\frac{dU(x,s)}{dx} - \frac{du(x,0)}{dx}$
$L^{-1}[F(s)G(s)] = \int_0^t f(t-u)g(u)du$	
$H(t-a)$	$\frac{e^{-as}}{s}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
$\sinh at$	$\frac{a}{s^2 - a^2}$

SMA840: METHODS OF APPLIED MATHEMATICS I

Purpose of the Course

To enable the learner to construct and solve mathematical models of physical problems using Partial Differential Equations.

Expected Learning Outcomes

By the end of the course the student should be able to:

i) Solve the Laplacian, the heat and wave equations.

ii) Solve the non-homogeneous problems.

iii) Solve the parabolic and hyperbolic equations in n- spaces

Course Content

Partial Differential Equations as mathematical models of physical problems. Linear second order equations and their classification (Laplace's equation, wave equation, heat-equation). Methods of solution to Green's function. Special analysis of elliptic differential operators in n-spaces.

Mode of Delivery

Lectures, Tutorials, Assignments, Seminars, Group work, Illustrations, Problem solving sessions.

Instructional Materials

Whiteboard, Chalkboard, Textbooks, Handouts, Lecture notes, LCD projector.

Course Assessment

Continuous Assessment Tests, Assignments, Presentations	40%
End of semester examination	60%
Total marks	**100%**

Peer Review

Monitoring by the head of department and fellow lecturers. Evaluation forms completed by students.

Core Reading Materials

1. Andres, V (2003). *Fourier Analysis and its Applications*, Springer-Verlag.
2. Evans, L.C. (1998). *PDEs*. Providence: AMS.
3. Grafakos, L (2008). *Modern Fourier Analysis*, Springer-Verlag.

Recommended Reading Materials

1. Cannon, J.R (1984). *The One Dimensional Heat Equation*. Reading: Addison-Wesley.
2. Friedman, A (2004). *PDEs of Parabolic Type*. Englewood Cliffs: Prentice Hall.
3. Folland, G.H. (1995*). Introduction to PDEs*, Princeton University Press.

Journals

1. IMA Journal of Applied Mathematics, Oxford Journals.
2. Indian Journal of Pure and Applied Mathematics, Springer.
3. Journal of Applied Mathematics, Hindawi Publishers.
4. Quarterly of Applied Mathematics, Brown University.