JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2ND YEAR 1ST SEMESTER 2016/2017 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAC 201

COURSE TITLE: FINANCIAL MATHEMATICS 1

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE ;

a).A woman takes out a home improvement loan for $11,000 over 5 years. She makes monthly payments in arrears and the bank charges an effective rate of interest of 6 % pa.

i.What is the monthly repayment [2 marks]
ii.How much interest does she pay on the third year? [2 marks]
iii.How much capital is repaid in the 20th installment [2 marks]
iv.At the end of the fourth year she decides to make further improvements to her house and wants to borrow another $ 4,000 at that stage. If her total balance is to be repaid over 3 years by level monthly payments and there is no alteration to the interest rate, how much is each payment? [3 marks]

b). $\delta(t) = \begin{cases} 0.05 + 0.02t for 0 \le t \le 5 \\ 0.15 \, for t > 5 \end{cases}$

i. Calculate the present value of $1000 due at the end of 12 [5 marks]

ii. Calculate the annual effective rate of discount implied by the transaction in (i) [2 marks]

c). Show that if d is the effective rate of discount per unit time, δ the constant force of interest per unit time and I the rate of interest per annum. Then

$$d^{(p)} = \delta - \frac{1}{2p}\delta^2$$

$$i^{(p)} = \delta + \frac{1}{2p}\delta^2$$ [6 marks]

d)An investor ,who has a sum of $ 10,000 to invest wishes to purchase an annuity certain with a term of 10 years. Calculate the amount of payments that can be provided if the annuity takes each of the following forms (assuming interest of 8 % pa effective)

(i)a level annuity payable monthly in arrears [3 marks]

(ii)a level annuity due payable half – yearly ,commencing in 2 years time [3 marks]

e)Distinguish between effective rate of interest and the nominal rate of interest [2 marks]

QUESTION TWO

a)On the basis of an interest rate of 12 % convertible quartely per annum . Find the present value of an annuity of $600 pa for 20 years payable [8 marks]

i. Annually in arrear

ii. Quarterly in arrear
iii. Monthly in arrear
iv. Continuously

b).A man borrowed kshs.7,500 to buy a car. He repays the loan by 24 monthly installments in arrears. The flat rate of interest is 9% pa.

i. What is his monthly payment [1 mark]
ii. What is the APR on this transaction [5 marks]

c).Your bank lends you some money for a year,and gives you the option of repaying with a single payment of $1200 at the end of the year ,or with two payments of $590 each,one in six months and the second at the end of the year.at what nominal interest rate convertible semi-annually, are those options equivalent [4 marks]

d)Given the nominal rate of interest of 18.5% per year, convertible monthly,calculate the nominal rate of discount convertible monthly [2 marks]

QUESTION THREE

1) A loan of $ 16,000 was issued to be repaid by level annuity certain payable annually in arrear over 10 years and calculated on the basis of an interest of 8 % per annum. The terms of the loan provided that any time the lender could alter the rate of interest, in which case the amount of the annual payment would be revised appropriately.

i. Find the initial amount of payment [4 marks]
ii. Immediately after the fourth payment was made the annual rate of interest as increased to 10 %.find the revised amount of level annual payment [4 marks]
iii. Immediately after the seventh payment was made the annual rate of interest was reduced to 9 %.There was no further change to the interest rate .Find the amount of the level payment and writes an expression for the entire transaction using the values you have worked out. [6 marks]

2. The force of interest $\delta(t)$ is a function of time and time t, measured in years is given by:

$$\delta(t) = 0.03 - 0.005t + 0.001t^2 \qquad 0 \le t \le 10$$

i. Calculate the equivalent constant force of interest per annum for the period t=0 to t=10 [3marks]
ii. Calculate the accumulated value at time t=7 of an investment of $250 at time t=0 plus a further investment of $150 at time t=5 [3marks]

QUESTION FOUR

a)A company is considering two capital investment projects .Project A requires an immediate expenditure of $1,000,000 and will produce returns of $ 270,000 at the end of each of the next 8 years .Project B requires an immediate investment of $ 1,200,000 together with a further expenditure of $

20,000 at the end of each of the first three years, will produce returns of $ 1,350,000 at the end of each of the sixth, seventh and eighth years

i)Calculate (to the nearest 0.1%) the IRR per annum for each project. [5 marks]

ii)Find the NPV of each project on the basis of effective annual interest rate of 15% [5 marks]

b)Two Projects A and B have the following expected cash flows:

	Project A	Project B
Initial Outlay	$170,000	$ 200,000
Other expenses	$20,000 at the end of year 1	-
	$ 10,000 at the end of year 2	-
Income	$20,000 at the end of year 1	$14,000 pa at the end of year of the first 6 years
	$20,000 at the end of year 2	$200,000 at the end of year 6
	$200,000 at the end of year 3	

(i) Calculate the internal rate of return (correct to 1 decimal place) for each project [4marks]

(ii) Calculate the net present value of each project using risk discount rate of 6 % pa [3marks]

(iii) If funds for the projects can be raised by borrowing from a bank, determine the interest rate charged by the bank above which each project becomes unprofitable. Mention any other factors that should be taken into account when deciding between the projects [3 marks]

QUESTION FIVE

a)Assume that $\delta(t)$,the force of interest per annum at time t (years) is given by the formula

$$\delta(t) = \begin{cases} 0.04, 0 < t \leq 1 \\ 0.05t - 0.01, 1 < t \leq 5 \\ 0.24, t > 5 \end{cases}$$

i)What is the total accumulated value at any time t(>0) of investment 1 at times 0,4 and 6 [6 marks]

ii)What is the present value at time 0 of a payment stream paid at a rate of : p(t)= 5t-1 received between time t=1 and t=5 [5 marks]

b)(i) Calculate the combined present value of an immediate annuity payable monthly in arrears such that payments are £ I ,000 pa for the first 6 years and £400 pa for the next 4 years, together with a lump sum of £2,000 at the end of the 10 years. [3 marks]

ii) Calculate the amount of the level annuity payable continuously for 10 years having the same present value as the payments in (i). [3 marks]

(iii) Calculate the accumulated values of the first 7 years' payments at the end of the 7th year for the payments in (i) and (ii). [3 marks]
Basis: Assume an interest rate of 12% pa convertible monthly