JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE IN BED SCI., BED ARTS AND BSC.

ACTURIAL SCIENCE

1st YEAR 2nd SEMESTER 2016/2017 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 102

COURSE TITLE: CALCULUS I

EXAM VENUE   **STREAM: BED SCI, BED ARTS AND BSC.**
                                        ACTUARIAL SCI.

DATE: **EXAM SESSION: TWO**

TIME: 2.00 HOURS

Instructions:

1. **Answer ONE (COMPULSORY) and any other two questions only.**

2. **Candidates are advised not to write on the question paper.**

3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

QUESTION ONE (30 MARKS)

a) Calculate the average rate of change of the function over the given interval (4 marks)
   i) $f(x) = 3x^2 + x^2 + 4$ $[2,3]$
   ii) $g(t) = \cot t$ $[^\pi/_6, ^{3\pi}/_4]$

b) Evaluate   (3 marks)

$$\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$$

c) Differentiate   (4 marks)
   i) $f(x) = \frac{x}{x-1}$
   ii) $y = 3x^2 \sin 2x$

d) A weight hanging from a spring is stretched down 5 units beyond its rest position and released at *t=0* to both up and down. Itsposition at any time t is $s = 5 \cos t$. What is its velocity and acceleration at *t=0*.   (4 marks)

e) Find $\frac{dy}{dx}$ if $y^2 = x^2 + \sin xy$ (5 marks)

f) Show that the point (2,4) lies on the curve $y^3 + x^3 - 9xy = 0$, then find the tangent and normal to the curve.   (5 marks)

QUESTION TWO (20 MARKS)

a) Find the slope of the curve at the point *P* and the equation of the tangent line at *P*. (4 marks)
   i) $y = 5 - x^2$ $P(1,4)$
   ii) $y = x^2 - 2x - 3$ $P(2,-3)$

b) Using the definition of limit, show that $\lim_{x \to 1}(5x - 3) = 2$ (3 marks)

c) Find the derivative of the function $y = x^3 + \frac{4}{3}x^2 - 5e^{ix} + 27$ (3 marks)

d) An object moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = \cos(t^2 + 1)$. Find the velocity of the object as a function of t.   (3 marks)

e) Find $\frac{dy}{dx}$ in: $y = \sin(x^2 + x)$.   (3 marks)

f) Prove that $\frac{dy}{dx}(\cos x) = -\sin x$ (4 marks)

QUESTION THREE (20 MARKS)

a) Consider the curve $3x^2 + xy + 4y^2 = 24$ (6 marks)
   i)   Find the formula in x and y for the slope of the tangent line at any *(x, y)* of the line.
   ii)   Write the slope intercept equation of the line tangent to the curve at the point (1, 6).
   iii)   Find the coordinates of the all other points on the curve where the slope of the tangent lies is same as tangent line at (1, 6).

b) Given $\lim\limits_{y \to t} g(y) \;=\; Q$ and $\lim\limits_{y \to t} f(y) \;=\; P$. Prove that $\lim\limits_{y \to t}(g(y) + f(y)) \;=\; Q + P$ (8 marks)

c) Find the derivative of: (6 marks)

       i) $y = \dfrac{4 \sin 5x}{5x^4}$

       ii) $y = (x^2 - \sin 2x)^4 \tan x$

QUESTION FOUR (20 MARKS)

a) Water runs into a conical flask at rate of $9m^3/_{min}$. The tank stands point down and has a height of 10m and the base radius is *5m*. How fast is the water rising when the water is *6m* deep.   (6 marks)

b) Using the first principles of differentiation, find the derivative of the cosine function. (4 marks)

c) Find the absolute maxima and minimum values of $g(t) = 8t^2 - t^4$ on $[-2, 1]$.   (3 marks)

d) Find y' in $y = \dfrac{\cos 2x}{1 - \sin\; 4x}$ (4 marks)

e) Evaluate $\lim\limits_{x \to \infty} \dfrac{2x + 5}{x^2 + 7x + 3}$ (3 marks)

QUESTION FIVE (20 MARKS)

a) Find $\int x \cos x \; dx$ (3 marks)

b) Show that $\int \cos^n x \; dx = \dfrac{\cos^{n-1} x \sin x}{n} + \dfrac{n-1}{n} \int \cos^{n-2} x \; dx$ (5marks)

c) Evaluate $\int x^2 e^x dx$ (3 marks)

d) Find the linearization of $f(x) = \sqrt{1 + x}$ at *x=3*   (4 marks)

e) Differentiate   (5 marks)

       i) $y = (2x + 1)^5$.

       ii) $r = 6(\sec\theta - \tan\theta)^{3/4}$.