**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE**

**IN APPLIED MATHEMATICS**

**2ND YEAR 1ST SEMESTER 2017 ACADEMIC YEAR**

**MAIN CAMPUS**

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**COURSE CODE: SMA 200**

**COURSE TITLE: CALCULUS II**

**EXAM VENUE:** **STREAM: (BSc. Actuarial)**

**DATE:** **EXAM SESSION:**

**TIME: 3.00 HOURS**

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**Instructions:**

1. **Answer ANY 3 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

## QUESTION ONE (COMPULSORY) (30 marks)

a) Find the antiderivative of the following expression:

$4x\left(2x^2+1\right)^4$ (4 marks)

b) Verify by differentiation that the formula is correct

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$ (5 marks)

c) Evaluate the definite integral

$\int_1^0 \left(3x^2+x-5\right)dx$ (4 marks)

d) Evaluate the indefinite integral below by using the given substitution to reduce the integral to standard form

$\int 12\left(y^4+4y^2+1\right)^2\left(y^3+2y\right)dy, \quad u=y^4+4y^2+1$ (4 marks)

e) Find the length of the curve $\frac{1}{3}\left(x^2+2\right)^{3/2}$ from $x=0$ to $x=3$ (5 marks)

f) Determine whether the following series converges or diverges

$\sum_{n=1}^{\infty}\frac{(-2)^n}{n!}$ (4marks)

g) By determining the area under the curve, $y=\frac{1}{2}x, \quad 0\le x\le 4$, show that the idea behind integration is that we can effectively compute many quantities by breaking them into small pieces, and then summing the contributions from each small part. (4 marks)

## QUESTION TWO (20 marks)

a) By completing the square and using appropriate substitution to reduce to standard form, evaluate the integral

$\int_2^4 \frac{2}{x^2-6x+10}dx$ (6 marks)

b) Using a substitution to reduce to standard form, evaluate

$\int_1^{e^{\pi/3}} \frac{dx}{x\cot(\ln x)}$ (4 marks)

c) By making the appropriate substitution for $u$:

(i) express the following integral in terms of $u$

(ii) evaluate the integral as function of $x$

$\int \frac{(x-2)^2}{\sqrt{2-x}}dx$ (6 marks)

d) By multiplying by a form of 1, evaluate the integral

$\int \frac{1}{\sec\theta-\tan\theta}d\theta$ (4 marks)

## QUESTION THREE (20 marks)

a) Express the integrand as a sum of partial fractions and evaluate the integral

$$\int_1^2 \frac{4y^2 - 7y - 12}{y(y+2)(y-3)} dy$$ (8 marks)

b) Evaluate the following integral by using a substitution prior to integration by parts

$$\int x^3 e^{5x} dx$$ (7 marks)

c) Obtain a reduction formula that expresses the integral $\int \sin^n x dx$ in terms of an integral of a lower power of $\sin x$. (5 marks)

## QUESTION FOUR (20 marks)

a) Find the volume of the solid generated by revolving the region bounded by the curve $y = x^2 + 1$ and line $y = x + 3$ about the $x$-axis. (7 marks)

b) Determine the area of the surface generated by revolving the curve $y = \sqrt{x+1}, 1 \le x \le 5$ about the $x$-axis. (6 marks)

c) Find the area of the region enclosed by the line $4x - y = 16$ and the curve $y^2 - 4x = 4$. (7 marks)

## QUESTION FIVE (20 marks)

a) Using eleven ordinates, apply Simpson's rule to evaluate the integral

$$4\int_0^1 \frac{dx}{1+x^2}$$ (5 marks)

b) Find a power series for the logarithmic function

$$L(x) = \ln(1+x)$$ (6 marks)

c) Show that the Taylor series about $x = 0$ for the function $f(x) = e^x$ is $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ .(5 marks)

d) Evaluate the following improper integral

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$$ (4 marks)