JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE(ACTUARIAL SCIENCE WITH IT)
3RD YEAR 2NDSEMESTER 2016/2017 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 312
COURSE TITLE: OPERATIONS RESEARCH I

EXAM VENUE: **STREAM: BED SCIENCE Y3S2**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE(30 MARKS)

a) Define the following terms used in linear programming
   i) Linear equation
   ii) Decision Variable
   iii) Linear programming problem
   iv) Sensitivity analysis (8 marks)

b) Define the following terms used in operations research
   i) Transportation problem
   ii) Assignment problem (4 marks)

c) .Use Gauss Jordan method to solve the set of simultaneous equations

$$2x_1 + x_2 - x_3 = 1$$
$$x_1 + 2x_2 + 2x_3 = 11 \qquad (6 \text{ marks})$$
$$2x_1 - 3x_2 + 4x_3 = 8$$

d) A school has to take 384 students on a tour at once. There are two types of buses available. Type A and Type B. Type A can carry 64 passengers each while type B can carry 48 passengers each. At least 7 buses have to be used.the school must not hire more than 5 buses of each type. The number of type A buses must be less than twice the number of type B buses hired while twice the number of type A buses must be more than the number of type B buses must be less than buses hired

The charges for hiring each type of bus are

Type A shs 25,000

Type B shs 20,000

(i) Formulate a linear programming problem hence state any three suitable methods that can be used in optimization. (7 marks)

(ii) Form a dual of the primal problem in (i) above. (4 marks)

QUESTION TWO (20 MARKS)

a) A firm produces two types of perfume A and B. It can produce 8 bottles of perfume every minute. The quantity of perfume A must be less than thrice the quantity of perfume B while twice the quantity of perfume A must exceed that of perfume B. Additional information is as follows.

	Production cost per bottle(kshs)	Labourers per bottle	Profit per bottle
A	300	1	70
B	100	4	50
Available	1800	20	

i) Develop a linear programming model based on the information above. (5 marks)

ii) Using graphical method and an isoprofit (search) line advise the firm on the number of bottles of each perfume to produce in order to maximize profit. (3 marks)

iii) Due to changing market forces the profit on perfume A increases to kshs 80 per bottle while that of B deacreses by 40% per unit. Determine whether the optimum level of production in b) ii) above will change and if it does what is the new optimum level of production. (4 marks)

b) Use the Dual simplex method to solve the following LP problem

Minimize $Z = 2x_1 + 3x_2 + 4x_3 + 5x_4$

Subject to

$$x_1 - x_2 + x_3 - x_4 \geq 10$$
$$x_1 - 2x_2 + 3x_3 - 4x_4 \geq 6$$
$$3x_1 - 4x_2 + 5x_3 - 6x_4 \geq 15$$
$$x_1, x_2, x_3, x_4 \geq 0$$

(8 marks)

QUESTION THREE(20 MARKS)

a) Use the Big M simplex method to

Maximize $Z = 22x_1 + 30x_2$

Subject to

$$3x_1 + 5x_2 \leq 130$$
$$-4x_1 + 5x_2 \geq 25$$
$$x_1 + 5x_2 \geq 75$$
$$x_1, x_2 \geq 0$$

(8 marks)

b) Suppose in a) above the left hand side values changed from 130 to 100, 25 to 58, and 75 to 125 respectively, determine the new level of optimum production. (6 marks)

c) Determine the range of values for which each coefficient of the objective function would lie in order to maintain the same optimum in a) above. (6 marks)

QUESTION FOUR (20 MARKS)

a) Use the dual of the LP problem below to solve it

Maximize $Z = 24x_1 + 21x_2 + 30x_3$

Subject to

$$12x_1 + 4x_2 + 8x_3 \le 240$$
$$8x_1 + 3x_2 + 3x_3 \le 140$$
$$6x_1 + 2x_2 + 3x_3 \ge 110$$
$$x_1, x_2, x_3 \ge 0$$

(10 marks)

b) A company has three ware houses A, B and C and four stores W, X, Y and Z. The warehouses have altogether a surplus of 1600 units of a given commodity as follows

A	300
B	800
C	500

The four stores together need a total of 1600 units of the commodity as follows

W	400
X	600
Y	400
Z	200

The cost of transporting one unit in Ksh from each warehouse to store is shown in the table below

	W	X	Y	Z
A	400	200	600	400
B	500	700	900	200
C	350	820	530	680

Determine which method would be cheaper and by how much between Vogel's approximation and the least cost cell method as far as the cost of transport is concerned (10 marks)

QUESTION FIVE (20 MARKS)

a) Consider the linear programming problem

Maximise $Z = c_1x_1 + c_2x_2 + c_3x_3$

Subject to

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix} x_1 + \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix} x_2 + \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix} x_3 + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} s_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} s_2 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} s_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} s_4 = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$x_1, \ldots\ldots\ldots\ldots s_4 \geq 0$

In the process of solving the problem, a tableau appears as follows

Basis	x_1	x_2	x_3	S_1	S_2	S_3	S_4	B
Z	0	-140	0	100	0	0	160	35400
x_3	0	$-\frac{1}{2}$	1	1	0	0	$-\frac{1}{2}$	18
S_2	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	0	0	11
S_3	0	2	0	$-\frac{1}{2}$	0	1	$-\frac{1}{2}$	16
x_1	1	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	31

i) Find the values of

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix}, \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \\ a_{42} \end{pmatrix}, \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \\ a_{42} \end{pmatrix} and \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

iii) Find the values of C_1 , C_2 and C_3 (12 marks)

b) Use the two phase method to solve the LP problem below

Maximize $Z = 2x_1 + 3x_2 + x_3$

Subject to

$$x_1 + x_2 + x_3 \leq 40$$
$$2x_1 + x_2 - x_3 \geq 10$$
$$-x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

(8 marks)