JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

1ST YEAR 2ND SEMESTER 2016/2017 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SMA 103

COURSE TITLE: LINEAR ALGEBRA I

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

a) Given that $\boldsymbol{u} = (1,2,-4)$, $\boldsymbol{v} = (-3,1,2)$ and $\boldsymbol{w} = (2,2,-4)$. Determine

i. $2\boldsymbol{v} + \frac{1}{2}\boldsymbol{w}$. (3mks)

ii. The dot product $\boldsymbol{v}.w$ (3mks)

b) Define the following terms:

i. Linearly independent vectors. (2mks)

ii. Basis for a vector space. (2mks)

c) Given that the vectors $\boldsymbol{u} = \begin{pmatrix} m \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ are perpendicular. Find m. (4mks)

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$.

i. Find the transpose of A. (1mk)

ii. Give the size of the matrices:

1) A (1mk)

2) B (1mk)

iii. Compute BA. (3mks)

d) Given that the matrix $\begin{bmatrix} 3 & 2 & 4 \\ 1 & x & 5 \\ 0 & 1 & -2 \end{bmatrix}$ is a singular matrix, determine the value of x.

(5mks)

e) Determine whether or not the vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ form a basis for $\mathbb{R}^2$. (5mks)

QUESTION TWO (20 marks)

a) Determine whether the following vectors are linearly dependent or linearly independent:

i. $\{(40,15), (-50, 25)\}$ (3mks)

ii. $v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $v_3 = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$

(4mks)

b) Determine characteristic polynomial, the eigenvalues and the associated eigenvectors of the matrix $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$. (10mks)

c) Let V and W be two vector spaces over a field F. Define a linear map f from V to W. (3mks)

QUESTION THREE (20 marks)

a) If $v_1, v_2, \ldots, v_n$ is a basis of the vector space V over a field K, then for every $v \in V$, then show that there are unique scalars $\lambda_1, \lambda_2, \ldots, \lambda_n$ such that
$v = \lambda_1 v_1 + \lambda_2 v_2 + \cdots + \lambda_n v_n$. (8mks)

b) Find the angle between the following vectors
$\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$. (6mks)

c) Use the Gauss method to solve each system and\ or conclude many solutions or no solution.

i.
$$2x + 2y = 5$$
$$x - 4y = 0$$

ii.
$$-x - y = 1$$
$$-3x - 3y = 2$$

. (6mks)

QUESTION FOUR (20 marks)

a) Given that $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \\ 2 & 3 & 1 \end{bmatrix}$, compute A^{-1}.

(8mks)

b) Use Gauss-Jordan reduction to solve the following system of linear equations

$$x + y + z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

(5mks)

c) Let V be a vector space over a field F.

i) Define a vector subspace U of V. (3mks)

ii) Show that y-axis in $\mathbb{R}^2$ is a vector subspace. (4mks)

QUESTION FIVE (20 marks)

a) Determine whether the space of rational numbers is a field or not. (10mks)

b) Let V be a vector space over a field F. Define a vector subspace U of V. (3mks)

c) Let V be a vector space. Prove that

i) There is a unique identity element; (3mks)

ii) There is a unique additive inverse for each element. (4mks)