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Functions of Multi-Pendula System in Spatial Motion

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Abstract— A study has been done using generalized coordinates system with the application of the Lagrangian formalism for n mass units linearly connected to move in space. Lagrangian equations have been developed and used throughout the research to determine the equations of motion for multi-pendula system for several as well as many mass units involved. These equations were solved using various mathematical techniques to determine the zenith angular accelerations for each system. The accelerations are found to be directly proportional to the sum of the product of the azimuth velocity and the respective zenith displacement.

Keywords—Lagrangian Formalism; Multi-Pendula System and Multi-Pendula; Zenith Angular Displacement; Zenith Angular Velocity.

I. Introduction

WHEN studying motions for n-tupple pendulum systems scholars used extensively the Newtonian to determine the dynamical variables. The limitations of this approach led to the introduction of the application of the Lagrangian methods. Generalized co-ordinate formalism gave easier computations of various quantities like velocity, force and momentum. Some studies have been carried out to obtain equations of motion and their solutions for multi- pendula systems using the Lagrangian formulations Spiegel (1967) and Chow (1995). Since then there is not much work done for motions of many-body pendulum systems (n-tupple pendula system where $n \geq 3$) fixed at one end and connected in series, one after another using an inextensible string. We carried out a study by Prichani et al., (2010) in which the zenith angular acceleration depends on its own angular displacement and the angular displacements of its nearest neighbors and later on we found that the motion of any mass was constrained in its motion by its nearest neighbors also. This is found in a paper by Sakwa et al., (2012).

A study of coupled systems was done by Hedrih [6-8]. He studied free vibrations of multi-pendula system inter-connected by light elements. The generalized forces corresponding to the generalized co-ordinates φ_k between two pendula are given by the equation.

$$\begin{aligned}\theta_{her(k)}(t) = p_{her(k,k+1)}(t) l_{T(k)} \\ - c_{T(k)} l_{T(k)}^2 (\varphi_{k+1} - \varphi_k)[1 \\ + \alpha_{T(k)} T_{(k)}(t)] \\ - c_{T(k)} \alpha_{T(k)} l_{T(k)} l_{0(k)} T_k\end{aligned} \tag{1}$$

A inverted chain of linked pendula balanced on top of one another was also studied by Achelson [1, 2] and Achelson & Mullin [3]. Pendulum systems extension were proposed and studied which include elastic and multi-body pendula model by Furuta et al., [4] and Spong et al., [17]. The initial conditions in the agreement protocol can be manipulated for results that satisfy linear constraints and also apply to the control of a distributed net-work of linearised pendula on a line graph topology [Nedic et al., 15]. Jones & Kush [9] gave more information about the examination of chaos variation in multiple pendulum systems with different amount of energy. They strongly speculated the complexity with the increase in mass units. Furuta et al., [5] successfully worked on the control of multi-pendula systems of varying constraints and further discussed the swing-up control of the pendulum units by considering the reach-ability of an unstable nonlinear control. A wave absorbing strategy was studied to suppress the vibration of a multiple-pendulum system. By this effective method, vibration of a multi-pendula system with arbitrary degrees of freedom can be controlled by measuring only the deflection angle of the uppermost pendulum adjacent to the support if its (pendulum) dimensions are known [Saigo et al., 16].

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Martin & Egerstedt [13] made effort to develop robotic marionettes systems which worked on discrete event systems. Maziar & Andre (2015) in Germany studied about the leg function and the ground reaction force in legged locomotion. They used the leg force modulated complaint hip in a new model for postural control in walking which employs the leg force feedback to adjust the hip compliance. This method gives a stable and robust walking in simulations and imitates human-like kinetic behavior. This gives the hip torque-angle relation for different walking speeds. The approach may physically implement the virtual pendulum concept in human animal locomotion. Tri-Nhut et al., [18] considered the demand for firemen and soldiers that could not be gain said for the need of safety and strategy in planning and co-ordination. They presented a new method to estimate both the forward displacement and orientation. The sensor unit is placed at the pedestrians ankles for the greater ease of usage. The 2D displacement is then computed based on the estimations for the pitch angle, yaw angle and the pedestrians leg length. This leg length is automatically estimated during the walking by exploiting the motion equation of a simple pendulum model and hence no prior measurements or training is required. This method employs the quaternion-based indirect kalman filter to estimate the Euler angles containing the yaw angle, the pitch angle and the roll angle. The real-time localization system has been implemented and experiments for various subjects conducted. Joot [10, 11] indicated that although setting up the Lagrangian, for a double pendulum was difficult, it was worse solving it. From his research he did not know what to expect in future. He avoided doing more work because it proved to be complicated. He further made clear that, calculating the energy explicitly for a general multiple pendulum was likely thought to be too pedantic for even the most punishing instructor to inflict on students as a problem or an example. This statement is what caused our anxiety and forms the basis of our work. In his other posting, it was noted that introducing any additional mass in system, for planer motion, the interaction coupling terms increase and thus complicate the kinetic energy specifications.

Martin et al., [12] used an inverted pendulum to calculate the length of gait in human locomotion. This is approximated by a simple pendulum where the influence of the knee and the ankle movements are not involved. The gait length is calculated from the extremes of vertical position of the human centre of mass during gait. The position of the centre of mass is calculated by double integration of acceleration which has errors. This article shows the influence of the inverted pendulum model as a methodology to measure the length of gait. Zhenglong & Nico [19] in their paper presented a framework for energy efficient dynamic human-like walk for a Nao humanoid robot. They used an inverted pendulum model to find an energy efficient stable walking gait. In this model they proposed a leg control policy which utilizes joint stiffness control. In order to identify the optimal parameters of the new gait for a Nao humanoid robot they used the policy gradient reinforcement. On testing this policy in a simulator and on a real Nao robot it was successful. It was shown that the new control policy had a dynamic walk that is more energy efficient than the standard walk of a Nao robot.

Mohamad et al., [14] studied the balance control of humanoid robots against external perturbations. The balancing actions are very demanding in terms of torque and power requirements for ankle joints. This is more so after strong and sudden impacts. An optimal control problem is formulated for the linearized inverted pendulum model to reduce the peak power requirements during ankle balancing strategy. This optimal control is calculated numerically and approximated by optimal compliance regulator whose ability is evaluated against other optimal methods. The stability is determined using the quadratic stability and parameter dependent Lyapunov functions. The efficacy of the proposed stabilizer validated for a complaint humanoid.

We have been able to give a quantitative study of a multi-pendula system in spatial motion through rigorous exercises by engaging Lagrangian formalism. There are unique possibilities of advancing studies in the n-tupple pendulum systems by the Lagrange method. This work covers all the aspects of suspended mass units oscillating in space. The approach is restricted to the derivations of equations of motion using the energy in the systems as opposed to the traditional application of force. The pendulum is used in science as a timing device and a tool in science education. In recent times it is being applied in robotic sciences. A multi-pendula system is one of the realities of a multi-component system subjected to motion and or vibrations.

II. FUNCTIONS OF MULTI-PENDULA SYSTEMS

A Lagrangian was developed to obtain equations of motion for dynamical systems by getting the difference between the kinetic energy and the potential energy of a multi-pendula system. This method is anchored on the determination of the degrees of freedom that would fit the situation. Generalized force can be related to the kinetic energy by the equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = \emptyset_i \qquad (2)$$

where $L = T - V$ is the Lagrangian and $\dot{\theta}_i$ & $\dot{\theta}_i$ are the generalized coordinates for position and momentum respectively. For the conservative systems $\emptyset_i = 0$, reducing equation (2) to

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = 0 \qquad (3)$$

Equations of motion depend on the degrees of freedom hence the need to use generalized Lagrangian to be applicable to as many masses as possible. This is intended to capture the various combinations of equal and unequal quantities of masses, lengths and the angles of inclination to the vertical.

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The kinetic energy and the potential energy terms for n mass units have the general equations as given in equation (4) and (5) respectively shown below.

$$T = \sum_{k=f(i,j)}^{n} m_k \left[\frac{1}{2}\sum_{i=1}^{n} l_i^2 \left\{\dot{\theta}_i^2 + \dot{\phi}_i^2 \sin^2\theta_i\right\} + l_i l_j \sum_{\substack{i=1 \\ i\neq j}}^{n}\left\{\dot{\theta}_i\dot{\theta}_j \cos(\theta_i - \theta_j) + \dot{\phi}_i\dot{\phi}_j \sin\theta_i \sin\theta_j\right\}\right] \quad (4)$$

Where i is the leading variable constant index and j is the trailing constant variable index.

$$V = \sum_{k=i}^{n} m_k \left(\sum_{i=1}^{n} l_i - \sum_{i=1}^{k} l_i \cos\theta_i\right) g \quad (5)$$

Equations (4) and (5) give the Lagrangian as

$$L = \sum_{k=f(i,j)}^{n} m_k \left[\sum_{i=1}^{n} \frac{l_i^2}{2}\left\{\dot{\theta}_i^2 + \dot{\phi}_i^2 \sin^2\theta_i\right\} + l_i l_j \sum_{\substack{i=1 \\ i\neq j}}^{n}\left\{\dot{\theta}_i\dot{\theta}_j \cos(\theta_i - \theta_j) + \dot{\phi}_i\dot{\phi}_j \sin\theta_i \sin\theta_j\right\}\right] - \sum_{k=i}^{n} m_k \left(\sum_{i=1}^{n} l_i - \sum_{i=1}^{k} l_i \cos\theta_i\right) g \quad (6)$$

The Lagrangian used in equation (6) generates equations of motion for n uneven quantities as given in equations (7), (8) and (9);

$$\begin{aligned}\ddot{\theta}_1 = {} & \frac{\left(\sum_1^n m_k\dot{\phi}_1 - \sum_2^n m_k\dot{\phi}_2\right)\dot{\phi}_1\theta_1}{m_1} + \frac{\sum_2^n m_k\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_2\theta_2}{m_1} \\ & + \frac{\sum_3^n m_k\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_3\theta_3}{m_1} \\ & + \frac{\sum_4^n m_k\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_4\theta_4}{m_1}\cdots \\ & \cdots\frac{\sum_{(n-2)}^n m_k\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_{(n-2)}\theta_{(n-2)}}{m_1} \\ & + \frac{\sum_{(n-1)}^n m_k\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_{(n-1)}\theta_{(n-1)}}{m_1} \\ & + \frac{m_n\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_n\theta_n}{m_1} \\ & - \frac{\left(\sum_1^n m_k\theta_1 - \sum_2^n m_k\theta_2\right)}{m_1}\frac{g}{l}\end{aligned} \quad (7)$$

$$\begin{aligned}\ddot{\theta}_{(n-1)} & \\ = {} & \left\{\frac{\left(\sum_{(n-1)}^n m_k\dot{\phi}_{(n-1)} - \sum_{(n-2)}^n m_k\dot{\phi}_{(n-2)}\right)}{m_{(n-2)}} + \frac{\left(\sum_{(n-1)}^n m_k\dot{\phi}_{(n-1)} - m_n\dot{\phi}_n\right)}{m_{(n-1)}}\right\}\dot{\phi}_1\theta_1 \\ & + \left\{\frac{\left(\sum_{(n-1)}^n m_k\dot{\phi}_{(n-1)} - \sum_{(n-2)}^n m_k\dot{\phi}_{(n-2)}\right)}{m_{(n-2)}} + \frac{\left(\sum_{(n-1)}^n m_k\dot{\phi}_{(n-1)} - m_n\dot{\phi}_n\right)}{m_{(n-1)}}\right\}\dot{\phi}_2\theta_2\cdots \\ & + \left\{\frac{\sum_{(n-1)}^n m_k\left(\dot{\phi}_{(n-1)} - \dot{\phi}_{(n-2)}\right)}{m_{(n-2)}} + \frac{\sum_{(n-1)}^n m_k\dot{\phi}_{(n-1)} - m_n\dot{\phi}_n}{m_{(n-1)}}\right\}\dot{\phi}_{(n-1)}\theta_{(n-1)} + \\ & \left\{\frac{m_n\left(\dot{\phi}_{(n-1)} - \dot{\phi}_{(n-2)}\right)}{m_{(n-2)}} + \frac{m_n\left(\dot{\phi}_{(n-1)} - \dot{\phi}_n\right)}{m_{(n-1)}}\right\}\dot{\phi}_n\theta_n \\ & - \left\{\frac{\sum_{(n-1)}^n m_k\theta_{(n-1)} - \sum_{(n-2)}^n m_k\theta_{(n-2)}}{m_{(n-2)}} + \frac{\sum_{(n-1)}^n m_k\theta_{(n-1)} - m_n\theta_n}{m_{(n-1)}}\right\}\frac{g}{l}\end{aligned} \quad (8)$$

$$\begin{aligned}\ddot{\theta}_n = {} & \left\{\frac{\left(m_n\dot{\phi}_n - \sum_{(n-1)}^n m_k\dot{\phi}_{(n-1)}\right)}{m_{(n-1)}} + \dot{\phi}_n\right\}\dot{\phi}_1\theta_1 \\ & + \left\{\frac{\left(m_n\dot{\phi}_n - \sum_{(n-1)}^n m_k\dot{\phi}_{(n-1)}\right)}{m_{(n-1)}} + \dot{\phi}_n\right\}\dot{\phi}_2\theta_2 \\ & + \left\{\frac{\left(\sum_{(n-1)}^n m_k\dot{\phi}_{(n-1)} - \sum_{(n-2)}^n m_k\dot{\phi}_{(n-2)}\right)}{m_{(n-1)}} + \dot{\phi}_n\right\}\dot{\phi}_3\theta_3\cdots\left\{\frac{\left(m_n\dot{\phi}_n - \sum_{(n-1)}^n m_k\dot{\phi}_{(n-1)}\right)}{m_{(n-1)}} + \dot{\phi}_n\right\}\dot{\phi}_{(n-2)}\theta_{(n-2)} \\ & + \left\{\frac{\left(m_n\dot{\phi}_n - \sum_{(n-1)}^n m_k\dot{\phi}_{(n-1)}\right)}{m_{(n-1)}} + \dot{\phi}_n\right\}\dot{\phi}_{(n-1)}\theta_{\{n-1\}} \\ & + \left\{\frac{m_n\left(\dot{\phi}_n - \dot{\phi}_{(n-1)}\right)}{m_{(n-1)}} + \dot{\phi}_n\right\}\dot{\phi}_n\theta_n \\ & - \left\{\frac{m_n\theta_n - \sum_{(n-1)}^n m_k\theta_{(n-1)}}{m_{(n-1)}} + \theta_n\right\}\frac{g}{l}\end{aligned} \quad (9)$$

A special case of n equal masses and distances gives the following equations:

$$\begin{aligned}\ddot{\theta}_1 = {} & \left\{n\dot{\phi}_1 - (n-1)\dot{\phi}_2\right\}\dot{\phi}_1\theta_1 + (n-1)\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_2\theta_2 \\ & + (n-2)\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_3\theta_3 + \cdots \\ & + 3\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_{(n-2)}\theta_{(n-2)} \\ & + 2\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_{(n-1)}\theta_{(n-1)} \\ & + \left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_n\theta_n \\ & - \left\{n\theta_1 - (n-1)\theta_2\right\}\frac{g}{l}\end{aligned} \quad (10)$$

$$\begin{aligned}\ddot{\theta}_{(n-1)} = {} & \left\{-3\dot{\phi}_{(n-2)} + 4\dot{\phi}_{(n-1)} - \dot{\phi}_n\right\}\dot{\phi}_1\theta_1 \\ & + \left\{-3\dot{\phi}_{(n-2)} + 4\dot{\phi}_{(n-1)} - \dot{\phi}_n\right\}\dot{\phi}_2\theta_2 \\ & + \left\{-3\dot{\phi}_{(n-2)} + 4\dot{\phi}_{(n-1)} - \dot{\phi}_n\right\}\dot{\phi}_3\theta_3\cdots \\ & + \left\{-3\dot{\phi}_{(n-2)} + 4\dot{\phi}_{(n-1)} - \dot{\phi}_n\right\}\dot{\phi}_{(n-2)}\theta_{(n-2)} \\ & + \left\{-2\dot{\phi}_{(n-2)} + 4\dot{\phi}_{(n-1)} - \dot{\phi}_n\right\}\dot{\phi}_{(n-1)}\theta_{(n-1)} \\ & + \left\{-\dot{\phi}_{(n-2)} + 2\dot{\phi}_{(n-1)} - \dot{\phi}_n\right\}\dot{\phi}_n\theta_n \\ & - \left\{-3\theta_{(n-2)} + 4\theta_{(n-1)} - \theta_n\right\}\frac{g}{l}\end{aligned} \quad (11)$$

$$\begin{aligned}\ddot{\theta}_n = {} & 2\left(\dot{\phi}_n - \dot{\phi}_{(n-1)}\right)\dot{\phi}_1\theta_1 + 2\left(\dot{\phi}_n - \dot{\phi}_{(n-1)}\right)\dot{\phi}_2\theta_2 \\ & + 2\left(\dot{\phi}_n - \dot{\phi}_{(n-1)}\right)\dot{\phi}_3\theta_3 + \cdots \\ & + 2\left(\dot{\phi}_n - \dot{\phi}_{(n-1)}\right)\dot{\phi}_{(n-2)}\theta_{(n-2)} \\ & + 2\left(\dot{\phi}_n - \dot{\phi}_{(n-1)}\right)\dot{\phi}_{(n-1)}\theta_{(n-1)} \\ & + \left(2\dot{\phi}_n - \dot{\phi}_{(n-1)}\right)\dot{\phi}_n\theta_n \\ & - 2\left(\theta_n - \theta_{(n-1)}\right)\frac{g}{l}\end{aligned} \quad (12)$$

If $n = 3$, for the special case of equal masses and distances, gives the following equations:

$$\begin{aligned}\ddot{\theta}_1 = {} & \left\{3\dot{\phi}_1 - 2\dot{\phi}_2\right\}\dot{\phi}_1\theta_1 + 2\left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_2\theta_2 \\ & + \left(\dot{\phi}_1 - \dot{\phi}_2\right)\dot{\phi}_3\theta_3 - \left\{3\theta_1 - 2\theta_2\right\}\frac{g}{l}\end{aligned} \quad (13)$$

$$\begin{aligned}\ddot{\theta}_2 = {} & \left\{-3\dot{\phi}_1 + 4\dot{\phi}_2 - \dot{\phi}_3\right\}\dot{\phi}_1\theta_1 + \left\{-2\dot{\phi}_1 + 4\dot{\phi}_2 - \dot{\phi}_3\right\}\dot{\phi}_2\theta_2 \\ & + \left\{-\dot{\phi}_1 + 2\dot{\phi}_2 - \dot{\phi}_3\right\}\dot{\phi}_3\theta_3 \\ & - \left\{-3\theta_1 + 4\theta_2 - \theta_3\right\}\frac{g}{l}\end{aligned} \quad (14)$$

$$\ddot{\theta}_3 = \{2(\dot{\phi}_3 - \dot{\phi}_2)\}\dot{\phi}_1\theta_1 + 2\{\dot{\phi}_3 - \dot{\phi}_2\}\dot{\phi}_2\theta_2 + \{2\dot{\phi}_3 - \dot{\phi}_2\}\dot{\phi}_3\theta_3 - 2\{\theta_3 - \theta_2\}\frac{g}{l} \qquad (15)$$

III. Discussion and Conclusion

The achievement is the determination of the Lagrangian general equation given by equation (6). It was possible to obtain from the general form in equation (10) the spatial motion for one mass whose equation was $\ddot{\theta} = \left(\dot{\phi}^2 - \frac{g}{l}\right)\theta$. The equations for the spatial motion of two unequal masses, at unequal lengths are determined and verified. There are equations also for equal quantities. The determination of the motion for three masses of both equal and unequal quantities was exclusively done and found agreable to the expectations. The angular accelerations are also determined. The other realization is for n unequal and equal mass units at different lengths of separation. The general relations for zenith angular accelerations $\ddot{\theta}_1$, $\ddot{\theta}_{(n-1)}$ and $\ddot{\theta}_n$ are stipulated in equations (7), (8) and (9) respectively. It is observed that the angular accelerations vary directly as the sum of the product of its zenith angular velocity and their respective A zenith angular displacements.

Matrix computations are used to get solutions for the angular accelerations in all the instances. The Gauss Sidel method is handy when two masses are involved. For three and four masses the application of the matrix inverse assists to get the solutions readily. The same method is applied to determine the general solutions for n mass units.

It might be quite interesting for one to embark on the determination of the Hamiltonian formulations for n suspended mass units set to oscillate in a plane and in space as well as the applicability of these findings especially in robotic science, theoretical physics and theoretical mechanics.

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