JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THEDEGREE OF BACHELOR OF EDUCATION (SCIENCE)

4TH YEAR 2ND SEMESTER

MAIN

REGULAR

COURSE CODE: SPH 403

COURSE TITLE: QUANTUM MECHANICS II

EXAM VENUE: **STREAM: (BED SCI)**

DATE: **EXAM SESSION:**

TIME: 2:00HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 (30 MARKS)

a)

i. State **TWO** postulates of Quantum mechanics**(1 mark)**

ii. Show that the solution of the time-dependent Schrödinger equation takes the form $\Psi(\vec{r},t)=\psi(\vec{r})u(0)e^{\frac{-i}{\hbar}Et}$ where the symbols have their usual meanings.**(3 marks)**

b) Using Dirac's notation of eigenfunctions, state the Riesz variational principle. **(1 mark)**

c) Define the following terms as used in Quantum mechanics.

i. Spin-orbit coupling**(1 mark)**

ii. Schrödinger picture**(1 mark)**

iii. Heisenberg picture**(1 mark)**

iv. Interaction picture **(1 mark)**

d) Show that the x-component of the orbital angular momentum is given by

$$\hat{L}_x = i\hbar\left(\sin\phi\frac{\partial}{\partial\theta}+\cos\phi\frac{\partial}{\partial\phi}\right)$$ **(4 marks)**

e) Show that the time-independent Schrödinger equation of a hydrogenic atom is given by $\left[-\frac{\hbar^2}{2\mu}\nabla^2-\frac{Ze^2}{4\pi\varepsilon_0 r}\right]\psi(\vec{r})=E\psi(\vec{r})$ where the symbols have their usual meanings. **(4 marks)**

f) Distinguish between time-independent perturbation theory and time-dependent perturbation theory. **(2 marks)**

g) State the selection rules for allowed transitions in hydrogen atom. **(2 marks)**

h) Account for the Pauli exclusion principle for fermions.**(2 marks)**

i) Derive the Heisenberg's equation of motion. **(4 marks)**

j) The spin-up and spin-down state vectors of an electron are respectively defined by $|u\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; |d\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Write down the Hermitian conjugate state vectors $\langle u|; \langle d|$ and show that they satisfy the orthonomality relations

$\langle u|u\rangle = \langle d|d\rangle = 1$; $\langle u|d\rangle = \langle d|u\rangle = 0$ **(3 marks)**

SECTION B

Attempt any TWO questions in this section.

QUESTION 2 (20 MARKS)

(i)Express the Hamiltonian of a one-dimensional linear harmonic oscillator in the form $\hat{H} = \hbar\omega\left(a^{+}a + \frac{1}{2}\right)$ where $\hat{a}$, $\hat{a}^{+}$ are the usual annihilation and creation operators which must be defined in the derivation. **(7 marks)**

(ii) Calculate the energy spectrum of the oscillator in the number state $|n\rangle$.

(4 marks)

(iii) Show that the ground state of the oscillator is a minimum uncertainty state, hence give the physical interpretation of such a state. **(9 marks)**

QUESTION 3 (20 MARKS)

a) (i) Matrix operators for the angular momentum operators can be defined by

$\hat{L}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$; $\hat{L}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$; $\hat{L}_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$. Determine the commutator $\left[\hat{L}_x, \hat{L}_y\right]$. **(3 marks)**

(ii) Show that the ladder operator $\hat{L}_+$, of the angular momentum in spherical coordinates takes the form $\hat{L}_+ = \hbar e^{i\phi}\left(\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right)$ **(7 marks)**

b) A particle moves in the one-dimensional potential defined by

$V(x)=\begin{cases} V_0\cos\left(\frac{\pi x}{2a}\right) & |x|\le a \\ \infty & |x|>a \end{cases}$. By treating the potential as a perturbation, obtain the first order energy correction, given that the unperturbed eigen function is

$$u_n=\begin{cases} \frac{1}{\sqrt{a}}\cos\left(\frac{n\pi x}{2a}\right) & n=odd \\ \frac{1}{\sqrt{a}}\sin\left(\frac{n\pi x}{2a}\right) & n=even \end{cases}.$$ **(10 marks)**

QUESTION 4 (20 MARKS)

a) Using $u(r)=rR(r)$, the radial equation for a one-electron atom is obtained in the form $\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2 l(l+1)}{2\mu r^2}-\frac{Ze^2}{4\pi\varepsilon_0 r}\right)u(r)=Eu(r)$ where the symbols have their usual meanings.

(i) By completing the square of the effective potential, determine the quantized orbit energy in the form $E_{l+1}=\frac{-\mu Z^2e^4}{2(4\pi\varepsilon_0)^2\hbar^2(l+1)^2}$. **(5 marks)**

(ii) Show that the radial equation can be factorized in the form

$\left(\frac{d}{dr}+K_{l+1}(r)\right)\left(-\frac{d}{dr}+K_{l+1}(r)\right)u(r)=\frac{2\mu}{\hbar^2}(E-E_{l+1})u(r)$ where the parameter $K_{l+1}(r)$ must be defined in the derivation. **(8 marks)**

b) Determine the highest quantized orbit solution of the factorized radial equation in a(ii) above in the form $u_n(r)=Ar^n e^{\frac{-Zr}{a_0 n}}: n=1,2,...$ where a_0 is the Bohr radius. **(7 marks)**

QUESTION 5 (20 MARKS)

a) A two-level system described by the wave function $\psi(t)=c_a(t)\psi_a e^{-iE_a\frac{t}{\hbar}}+c_b(t)\psi_b e^{-iE_b\frac{t}{\hbar}}$ experiences a time-dependent perturbation. Suppose the system is in state ψ_a intially, derive the expressions for the first order approximations of probability amplitudes, $c_a^1(t)$ and $c_b^1(t)$. **(12 marks)**

b) If the perturbation in 5 (a) above is of the form $H_{ab}^I = V_{ab}\cos\omega t$, show that the transition probability is given by $P_{a\to b} = \frac{|V_{ab}|^2}{\hbar^2}\frac{\sin^2(\omega_0-\omega)t}{(\omega_0-\omega)^2}$ **(8 marks)**