JARAMOGI OGINGA ODINGA UNIVERSITY
OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION 2013/2014

3RD YEAR 1ST SEMESTER EXAMINATION FOR THE

DEGREE OF BACHELOR OF EDUCATION (SCIENCE) WITH

IT

(SCHOOL BASED-MAIN)

COURSE CODE: SMA 301

TITLE: ORDINARY DIFFERENTIAL EQUATIONS
DATE: 2/5/2013 TIME: 9.00-11.00AM

DURATION: 2 HOURS

INSTRUCTIONS

1. **This paper contains FIVE (5) questions**
2. **Answer question 1 (Compulsory) and ANY other 2 Questions**
3. **Write all answers in the booklet provided**

QUESTION ONE (COMPULSORY)

a) Given $y = A\sin x - B\cos x$, where A and B and arbitrary constants, eliminate the arbitrary constants to form a differential equation hence state its order and degree (5 marks)

b) The rate of cooling of a body is proportional to the excess of its temperature above its surrounding $^{-0}$C. A body cools from 85^{0}C to 65^{0}C in 4.0 minutes at a surrounding temperature of 15^{0}C. Determine how long to the nearest second the body will take to cool to 55^{0}C. (4 marks)

c) Solve the differential equation below using an appropriate method

$$\frac{d^2y}{dx^2} + 9y = 0$$ (5 marks)

d) Using an appropriate method solve the differential equation $2yy'' = 1 + (y')^2$. (5 marks)

e) Use the method of variation of parameters to solve $2\frac{d^2y}{dx^2} - 7\frac{dy}{dx} - 4y = e^{3x}$. (5 marks)

f) Solve the differential equation $(y - x - 4)dy + (2 - y - x)dx = 0$ (6 marks)

QUESTION TWO (20 marks)

a) By finding the integrating factor , find the general solution of the differential equation $\frac{(1-x^2)}{x}\frac{dy}{dx} + \frac{2x^2 - 1}{x^2}y = x$ (Hint: Use partial fractions) (10 marks)

b) A resistance (R) of 100 ohms, an inductance (L) of 0.5 henry are connected in series with a battery of 20 volts(V). Find the current (i) in the circuit as a function of time(t) given that they are connected by the differential equation $Ri + L\frac{di}{dt} = V$. (5 marks)

c) Use variation of parameters to solve the differential equation

$$4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + e^x = 0.$$ (5 marks)

QUESTION THREE (20 marks)

a) Consider a second order differential equation

$$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=F(x),$$

Let F(x) = 0 and let y = U and y = V , where U and V are functions of *x* be two solutions to the differential equation, then show that y = (U + V) is also a solution. (6 marks)

b) Find the general solution of the differential equations

(i) $\left(\sqrt{xy}-x\right)dy+ydx=0$ (4 marks)

(ii) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=0$ (4 marks)

(iii) $\frac{d^2y}{dx^2}-36y=2\cos 4x$ (4 marks)

QUESTION FOUR (20 marks)

Use any appropriate method to solve each of the differential equations below

a) $\left[(\cos x)In(2y-8)+\frac{1}{x}\right]dx=\frac{\sin x}{4-y}dy$ given that *y* = 4.5 when *x* = 1. (6 marks)

b) $yy''+(y')^2=0$ (6 marks)

c) $\frac{dx}{dy}+\frac{y}{1-y^2}x=y\sqrt{x}$ (8 marks)

QUESTION FIVE (20 marks)

a) Detectives discovered a murder victim at 6.30 am and the body temperature of the victim was then 26 ^{0}C. After 30 minutes the police surgeon arrived and found the body temperature to be 23 ^{0}C. If the air temperature was 16 ^{0}C throughout and the normal body temperature is 37 ^{0}C. At what time did the police surgeon estimate that the crime occurred. (10 marks)

b) Solve the differential equation $xy''=y'+(y')^3$ given $x=1$ when $y=1$ and $x=2$ when $\frac{dy}{dx}=1$ (10 marks)