JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

4^{th} ***YEAR*** 1^{st} ***SEMESTER EXAMINATION*** [SCHOOL BASED]

SMA 403: TOPOLOGY

INSTRUCTION: Attempt question one (**COMPULSORY**) and any other TWO questions only.

QUESTION ONE(*COMPULSORY*) [30 MARKS]

(a). Define the following terms: Open set, closed set, boundary point and interior point. (5 marks)

(b). Let X be a topological space. Prove that a subset V of X is open in X iff V is a neighbourhood of each point belonging to V. (5 marks)

(c). Distinguish between a topological subspace and a basis of a topological space. (4 marks)

(d). Define a homeomorphism giving an example. (4 marks)

(e). If $L = \{(x, y) \in \mathbb{R}^n : 7x + 3y = 14\}$ and $r = (2, -1)$, find the distance between the point r and L. (4 marks)

(f). Let X be a topological space, and let A be a subset of X. A subset B of A is closed in A iff $B = A \setminus F$ for some closed subset F of X. (5 marks)

(g). Given the set $V = \{v : v \; is \; a \; digit\}$, find the cardinality of V and the subset of V containing nonzero prime numbers. (3 marks)

QUESTION TWO [20 MARKS]

(a). Define a topological space. (3 marks)

(b). Let $P = \{1, 2, 3\}$, $\hbar = \{\emptyset, \{1\}, \{1, 2\}, \{1, 3\}, P\}$ and $\hbar' = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, P\}$. Determine whether $\hbar$ and $\hbar \cup \hbar'$ are topologies on P. (17 marks)

QUESTION THREE [20 MARKS]

(a). Define a Hausdorff space giving one example. (5 marks)

(b). Prove that all metric spaces are Hausdorff spaces. (15 marks)

QUESTION FOUR [20 MARKS]

(a). Define continuity of a function between topological spaces. (2 marks)

(b). Let X, Y, Z be topological spaces, and let $f : X \to Y$ and $g : Y \to Z$ be continuous functions. Prove that the composition $g \circ f : X \to Z$ of the functions f and g is continuous. (14 marks)

(c). Let X, Y be topological spaces, and let $f : X \to Y$ be a function from X to Y. Prove that the function f is continuous if and only if $f^{-1}(G)$ is closed in X for every closed subset G of Y. (4 marks)

QUESTION FIVE [20 MARKS]

(a). Define a metric space. (3 marks)

(b). If $X = \{(x, y) \in \mathbb{R}^2 : xy = 1\}$ and $Y = \{(x, y \in \mathbb{R}^2 : y = 0)\}$, find $d(X, Y)$. (2 marks)

(c). Let $\mathbb{R}$ be the set of of real numbers. Show that $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by $d(x, y) = |x - y|$, for all $x, y \in \mathbb{R}$ a metric on $\mathbb{R}$. (9 marks)

(d). Describe two applications of the study of topology to real life situations giving relevant examples. (6 marks)