



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY  
SCHOOL OF HEALTH SCIENCES**

**UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR  
OF SCIENCE (ACTUARIAL SCIENCE)**

**1<sup>ST</sup> YEAR 2<sup>ND</sup> SEMESTER 2013/2014 ACADEMIC YEAR**

**CENTRE: MAIN**

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**COURSE CODE: SAC 102**

**COURSE TITLE: FUNDAMENTALS OF ACTUARIAL MATHEMATICS I**

**EXAM VENUE: LR I**

**STREAM: BSc. (Actuarial)**

**DATE: 13/12/2013**

**EXAM SESSION: 2.00 – 4.00 PM**

**TIME: 2 HOURS**

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**Instructions:**

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE**

a).(i) State the three methods of graduation. [3 marks]

(ii) State at least two advantages and two disadvantages of the three methods of graduation

[9 marks]

b) Suppose that  $s(x) = (1 - x/\bar{S})^r$

Give simple formulae for

i.  $\bar{a}_x$

ii.  $e_x^0$

iii.  ${}_{10}P_{70}$

iv.  ${}_{40/5}q_{35}$

[8 marks]

(iii)What are the aims of a graduation [3 marks]

b).Define the terms selection and underwriting [2 marks]

c) Calculate the variance ratio for an investigation based on the observation of the policies in the following table, using the binomial model of mortality.

Number of policies,n	1	2	3	4	5
Number of lives holding ,n policies	1,107	62	16	0	2

In addition, calculate the expectation and variance of the number of policies becoming claims by death if policies are held by 50 year olds with  $q_{50} = 0.00534$  . [5marks]

**QUESTION TWO [20 marks]**

a) A mortality table is defined such that

$${}_t p_x = \left(1 - \frac{t}{110 - x}\right)^{\frac{1}{2}} \quad \text{for } x < 110, t < 110 - x$$

And  ${}_t p_x = 0$ , for  $t \geq 110-x$

Calculate:

- i. The complete expectation of life at exact age 45 [3 marks]
  - ii. The force of mortality at age 45 [3 marks]
- b) State the characteristics of a good graduation [3 marks]
- c). (b) For a certain population  $l_x = 10,000(121 - x)^{\frac{1}{2}}$  [5 marks]
- Find  $u_x, q_x$  and the probability that a life aged 0 will die between ages 21 and 40
- d) For a particular population  $e_{45} = 40.20$  and  $e_{46} = 39.27$ . Calculate  $q_{45}$  [3 marks]
- e). Show that  ${}_{m/n} q_x = {}_m p_x - {}_{m+n} p_x$  [3 marks]

### **QUESTION THREE [20 marks]**

- a.) Define the following terms briefly [4 marks]
- i. Temporary initial selection
  - ii. Self selection
  - iii. Reverse selection
  - iv. Adverse selection (or anti- selection)
- b) How does the central exposed to risk differ from the initial exposed to risk? [3 marks]
- c). Prove each of the following results:
- (i) The probability that exactly  $x$  decrements will occur in a population consisting initially of  $n$  individuals subject to a single decrement with rate  $q$  per annum is
- $$\binom{n}{x} q^x (1 - q)^{n-x} \quad [4 \text{ marks}]$$
- (ii) The maximum likelihood estimator of the parameter  $q$  for the binomial model equals the number of decrements divided by the initial population. [5 marks]
- (iii) The maximum likelihood estimator of the parameter  $\lambda$  for the Poisson model is an unbiased estimator of the force of decrement. [4 marks]

#### **QUESTION FOUR**

- a) State the assumptions underlying the binomial mortality model [2 marks]
- b) A cat has nine lives, so the cat will not die until it has lost all nine of its lives. The probability of a cat losing a life is 20 % per week. Assuming that the mortality of each life follows the binomial model , calculate the following
- (i) The probability that a cat who currently lost none of its nine lives will die during the next 10 weeks . [4marks]
- (ii)The probability that this cat will die during the fifth week . [4 marks]
- c) A large life insurance company is investigating its mortality experience over the period 1 January 2005 to 30 June 2008.

Censuses of the number of in force policyholders classified by age next birthday at entry plus nearest duration at the census date are available at each 1 January from 2005 to 2008 and also at 30 June 2008. The numbers of deaths classified by age next birthday at entry plus curtate duration at date of death are available for each inter-censal period.

- (i)State, with reasons, the rate you interval you would use for this investigation. State the age definition you would use , and indicate the exact age to which your estimated initial rates of mortality would apply. [4 marks]
- (ii)Derive an expression for the initial exposed to risk corresponding to your answer to (i) .You may also assume that 30 June occurs exactly halfway through the calendar year .All other assumptions must be stated. [6 marks]

#### **QUESTION 5 [20 marks]**

- i) Describe the process of carrying out a graduation using the graphical method. Your description . should cover the three main steps of grouping sparse data, sketching the curve and hand polishing. [5 marks]
- ii) Discuss the advantages and disadvantages of using the graphical method of graduation, explaining under which circumstances it is appropriate to use it. Describe the situations where it is not an appropriate method. [8 marks]
- iii)What are the aims of a graduation. [3 marks]
- iv).Define the terms selection and underwriting. [4 marks]