

JAROMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

BSc.ACTUARIAL SCIENCE

2ND YEARS FIRST SEMESTER

COURSE NAME : FINANCIAL MATHEMATICS

COURSE CODE : SAC 201

ANSWER QUESTION 1 (COMPULSORY) AND ANY OTHER TWO.

QUESTION ONE ;

1.a).A woman takes out a home improvement loan for \$11,000 over 5 years. She makes monthly payments in arrears and the bank charges an effective rate of interest of 6 % pa.

- i. What is the monthly repayment [2 marks]
- ii. How much interest does she pay on the third year? [2 marks]
- iii. How much capital is repaid in the 20th installment [2 marks]

b)At the end of the fourth year she decides to make further improvements to her house and wants to borrow another \$ 4,000 at that stage. If her total balance is to be repaid over 3 years by level monthly payments and there is no alteration to the interest rate, how much is each payment? [3 marks]

(c) Assume that

$$u(t) = \begin{cases} 0.05 + 0.02t & \text{for } 0 \leq t \leq 5 \\ 0.15 & \text{for } t > 5 \end{cases}$$

- i. Calculate the present value of \$1000 due at the end of 12 [5 marks]
- ii. Calculate the annual effective rate of discount implied by the transaction in (i) [2 marks]

(d). Show that if d is the effective rate of discount per unit time, u the constant force of interest per unit time and i the rate of interest per annum. Then

$$d^{(p)} = u - \frac{1}{2p}u^2$$

$$i^{(p)} = u + \frac{1}{2p}u^2 \quad [6 \text{ marks}]$$

(e). If $u(t)$ and $A(t_0, t)$ are continuous functions for $t \geq t_0$, and the principle of consistency holds, then for $t_0 \leq t_1 \leq t_2$. Show that :

$$A(t_1, t_2) = \exp \int_{t_1}^{t_2} u(t) dt \quad [8 \text{ marks}]$$

QUESTION TWO

a) On the basis of an interest rate of 12% per annum effective. Find the present value of an annuity of \$600 pa for 20 years payable [8 marks]

- i. Annually in arrear
- ii. Quarterly in arrear
- iii. Monthly in arrear
- iv. Continuously

b). A man borrowed kshs. 7,500 to buy a car. He repays the loan by 24 monthly installments in arrears. The flat rate of interest is 9% pa.

- i. What is his monthly payment [1 mark]
- ii. What is the APR on this transaction [5 marks]

c). The force of interest $u(t)$ is a function of time and time t , measured in years is given by:

$$u(t) = 0.03 - 0.005t + 0.001t^2 \quad 0 \leq t \leq 10$$

- i. Calculate the equivalent constant force of interest per annum for the period $t=0$ to $t=10$ [3marks]
- ii. Calculate the accumulated value at time $t=7$ of an investment of \$250 at time $t=0$ plus a further investment of \$150 at time $t=5$ [3 marks]

QUESTION THREE

(a) A loan of \$ 16,000 was issued to be repaid by level annuity certain payable annually in arrear over 10 years and calculated on the basis of an interest of 8 % per annum. The terms of the loan provided that any time the lender could alter the rate of interest, in which case the amount of the annual payment would be revised appropriately.

- i. Find the initial amount of payment [4 marks]
- ii. Immediately after the fourth payment was made the annual rate of interest as increased to 10 %. Find the revised amount of level annual payment [4 marks]
- iii. Immediately after the seventh payment was made the annual rate of interest was reduced to 9 %. There was no further change to the interest rate. Find the amount of the level payment and writes an expression for the entire transaction using the values you have worked out. [6 marks]

(b). The force of interest $u(t)$ is a function of time and time t , measured in years is given by:

$$u(t) = 0.03 - 0.005t + 0.001t^2 \quad 0 \leq t \leq 10$$

- i. Calculate the equivalent constant force of interest per annum for the period $t=0$ to $t=10$ [3marks]
- ii. Calculate the accumulated value at time $t=7$ of an investment of \$250 at time $t=0$ plus a further investment of \$150 at time $t=5$ [3marks]

QUESTION FOUR

a) The following data relates to the assets of an investment fund

date	Market value
1 January 2002	\$4.2 m
1 January 2003	\$4.6m
1 January 2004	\$5.1m
1 July 2004	\$5.1 m
31 December 2004	\$5.5 m

The only cash flow during the calendar years 2002,2003 and 2004 that was not generated from the assets of the fund was a payment of \$800,000 ,received by the fund on 30 June 2004.

For the period 1 January 2002 to 31 December 2004: Calculate

- i. The money weighted rate of return
- ii. The time weighted rate of return
- iii. The linked rate or return (using equal year-long linking periods)

Express your answers as annual rates rounded to the nearest 0.1 % [8 marks]

b)A second fund was found to have a money weighted rate of return of 3.5 % pa and a time weighted rate of return of 3.5 %.Compare the relative performance of two fund managers [2 marks]

(c) Two Projects A and B have the following expected cash flows:

	Project A	Project B
Initial Outlay	\$170,000	\$ 200,000
Other expenses	\$20,000 at the end of year 1	-
	\$ 10,000 at the end of year 2	-
Income	\$20,000 at the end of year 1	\$14,000 pa at the end of year of the first 6 years
	\$20,000 at the end of year 2	\$200,000 at the end of year 6
	\$200,000 at the end of year 3	

- (i) Calculate the internal rate of return (correct to 1 decimal place) for each project [4marks]
- (ii) Calculate the net present value of each project using risk discount rate of 6 % pa [3marks]
- (iii) If funds for the projects can be raised by borrowing from a bank, determine the interest rate charged by the bank above which each project becomes unprofitable. Mention any other factors that should be taken into account when deciding between the projects [3 marks]

QUESTION FIVE

Assume that $u(t)$, the force of interest per annum at time t (years) is given by the formula

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leq t < 5 \\ 0.06 & \text{for } 5 \leq t < 10 \\ 0.04 & \text{for } t \geq 10 \end{cases}$$

a.) Derive expressions for $v(t)$, the present value of 1 due at time t . [8 marks]

b) An investor effects a contract under which he will pay 15 premiums annually in advance into an account which will accumulate according to the above force of interest. Each premium will be of amount kshs. 600 and the first premium will be paid at time 0. In return the investor will receive either :

- i. The accumulated amount of the account one year after the final premium is paid ; or
- ii. A level annuity payable annually for 8 years, the first payment being made one year after the final premium is paid.

Find the lump sum payment under option (i) and the amount of annual annuity under option (ii)

[12 marks]

