

JAROMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

BSc.ACTUARIAL SCIENCE

3RD YEARS SECOND SEMESTER

COURSE NAME: FINANCIAL MATHEMATICS 2

COURSE CODE: SAC 300

ANSWER QUESTION 1 (COMPULSORY) AND ANY OTHER TWO.

QUESTION 1 [30 marks]

a) An insurance company has liabilities consisting of eleven annual payments of £1 million, with the first payment due to be made in 10 years' time and the last payment due to be made in 20 years' time. The rate of interest is 6% per annum effective.

- i. Show that the discounted mean term of these liabilities, to four significant figures, is 14.42 years
[6 marks]

The insurance company holds two zero-coupon bonds, one paying £X in 10 years' time and the other paying £Y in 20 years' time.

- ii. Find values of X and Y such that Redington's first two conditions for immunisation from small changes in the rate of interest are satisfied. [8 marks]
- iii. Explain, without making any further calculations, whether you would expect Redington's third condition for immunisation to be satisfied for the values of X and Y calculated in ii. above [4 marks]

b) Consider a portfolio consisting of two securities A and B. you have the following information about the securities

.The return on A is equally likely to be 5% or 10 %

. The return on B is equally likely to be 10% or 20 %

- i. Calculate the mean and variance of returns on each individual security [4 marks]
- ii. Calculate the mean and variance of the return of the portfolio as a whole given that correlation coefficient of the two securities is: [6 marks]
- a. 1
b. 0
c. -1
d. 0.7

(iii) Comment on your results

[2 marks]

QUESTION 2 [20 marks]

1. Define the following measures of investment risk

[8 marks]

- a) Variance of return
b) Downside semi-variance of return
c) Shortfall probability
d) Values at risk

2. 1. An investor is contemplating an investment with a return of \$ R, where:

$$R = 300,000 - 500,000U$$

Where U is a uniform [0,1] random variable

Calculate each of the following four measures of risk:

[12 marks]

- a) Variance of return
- b) Downside semi- variance of return
- c) Shortfall probability, where shortfall level is \$ 100,000
- d) Value at risk at the 5% level

QUESTION 3 [20 marks]

- a) State the assumptions of mean –variance portofolio theory [7 marks]
- b) State the properties of a random walk [5 marks]
- c) You are given that assets X and Y are perfectly correlated such that $R_Y = 6 + 0.2 R_X$ and the probability distribution of X is

| Probability | Return on X, R_X |
|-------------|--------------------|
| 0.1 | 30 |
| 0.2 | 20 |
| 0.4 | 15 |
| 0.2 | 10 |
| 0.1 | -50 |

- a) Graph the portfolio opportunity set [3marks]
- b) What is the percentage of your wealth to put into asset X to achieve zero variance? [5 marks]

QUESTION 4 [20 marks]

A pension fund has the following liabilities: annuity payments of £160,000 per annum to be paid annually in arrears for the next 15 years and a lump sum of £200,000 to be paid in ten years. It wishes to invest in two fixed-interest securities in order to immunize its liabilities. Security A has a coupon rate of 8% per annum and a term to redemption of eight years. Security B has a coupon rate of 3% per annum and a term to redemption of 25 years. Both securities are redeemable at par and pay coupons annually in arrears.

- (i) Calculate the present value of the liabilities at a rate of interest of 7% per annum effective. [5 marks]
- (ii) Calculate the discounted mean term of the liabilities at a rate of interest of 7% per annum effective. [5 marks]
- (iii) Calculate the nominal amount of each security that should be purchased so that both the present value and discounted mean terms of assets and liabilities are equal. [8 marks]
- (iv) Without further calculation, comment on whether, if the conditions in (iii) are fulfilled, the pension fund is likely to be immunized against small, uniform changes in the rate of interest. [2 marks]

QUESTION 5 [20 MARKS]

(i) In any year, the interest rate per annum effective on monies invested with a given bank has mean value j and standard deviation s and is independent of the interest rates in all previous years.

Let s_n be the accumulated amount after n years of a single investment of 1 at time $t = 0$.

(a) Show that $[E (s_n)] = (1 + j)^n$ [4 marks]

(b) Show that $\text{Var} [s_n] = (1 + 2j + j^2 + s^2)^n - (1 + j)^{2n}$ [6 marks]

(ii) The interest rate per annum effective in (i), in any year, is equally likely to be i_1 or i_2 ($i_1 > i_2$). No other values are possible.

(a) Derive expressions for j and s^2 in terms of i_1 and i_2 [4 marks]

(b) The accumulated value at time $t = 25$ years of £1 million invested with the bank at time $t = 0$ has expected value £5.5 million and standard deviation £0.5 million.

Calculate the values of i_1 and i_2 [6 marks]