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UNIVERSITY EXAMINATIONS 2012/2013

THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE WITH IT

SAC 305: PENSION MATHEMATICS

INSTRUCTIONS: Attempt Question ONE and ANY OTHER TWO questions

QUESTION 1 [COMPULSORY] [30 Marks]

(a) State giving the features, the types of pension schemes [**6 Marks**]

(b) The valuation assumptions for a pension scheme are $i = 9\%$, and $e = 7\%$. What is the percentage change in the value of liabilities, with a mean term to retirement of 15 years if:

(i) i and e are both reduced by 1%p.a?   [**2 Marks**]

(ii) i and e are both increased by 3%p.a?   [**2 Marks**]

(iii) i is unchanged but e is reduced by 1%?   [**2 Marks**]

(c)(i) What is the main factor that affects the risk that a pension guarantee is not met?   [**3 Marks**]

(ii) Give three examples of options that may be given to an individual in regard to pension scheme benefits.   [**3 Marks**]

(d) A portfolio might consist of:

<div align="center">

Domestic equity 55%

Overseas equity 20%

Property 3%

Index-linked bonds 7%

Fixed interest bonds 15%

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State two possible homogenous grouping of the above assets. [**4 Marks**]

(e) The value of a pension P payable on ill-health retirement to an individual currently aged x could be expressed as

$$P \sum_{y=x}^{64} v^{y+\frac{1}{2}-x} \frac{i_y}{l_x} a^i_{y+\frac{1}{2}} = \frac{P}{D_x} \sum_{t=0}^{64-x} C^{ia}_{x+t}$$

$$= P \frac{M^{ia}_x}{D_x}$$

(i) Define C^{ia}_{x+t} and M^{ia}_x as used above. **[2 Marks]**

(ii)Calculate the present value of an annuity of Kshs.1, 000, 000 p.a payable on retirement at any age up to 65, due to;

(a)ill-health retirement. **[3 Marks]**

(b) normal retirement **[3 Marks]**

Assume the member is currently aged 50.

QUESTION 2[20 MARKS]

(a) Using the following assumptions and data $i = 10\%$, $e = 8\%$, $A = 60$, $R = 65$ and $a'_R = 12$, calculate the AL and SCR under the Projected Unit for the following members individually;

(i) 25 year old, no past service, salary Kshs.20, 000 **[4 Marks]**

(ii) 40 year old, 15 years past service, salary Kshs.15, 000 **[4 Marks]**

(iii) 55 year old, 30 years past service, salary Kshs.30, 000 **[4 Marks]**

The same earnings definition is used for benefit purposes. Assume for simplicity, that contributions are paid continuously and salary growth is continuous.

(b) You are the actuary to a pension scheme which has only two members. One of the members is aged 20 and has recently joined the scheme with a pensionable salary of Kshs.10, 000. The other member is aged 59 and has a pensionable salary of Kshs.100, 000.

You have been asked to recommend a contribution rate for this scheme.Currently the scheme has no surplus or deficit if the PU method is used. Calculate the PUSCR and AASCR, for this data using the following assumption data and comment on your answer.

$$i = 9\%$$

$$e = 7\%$$

$$A = 60\%(\text{ie accrual rate is 60ths})$$

$$R = 60$$

$$a'_R == 15$$

[**8 Marks**]

QUESTION 3[20 MARKS]

(a) (i) Define 'actuarial funding' as applied in actuarial valuation. [**3 Marks**]

(ii) Describe the possible alterations to the Current Unit (CU) Method of actuarial valuation and the relationship of the resultant method and other methods. [**4 Marks**]

(b) Consider the general equation for the Standard Contribution Rate (SCR) given by

$$SCR = \frac{f \times Y \times S \times \frac{r_{65}}{l_x}(1+r)^{65-x-Y}(1+e)^Y v^{65-x} a^r_{65} - AL \times \left(\frac{(1+e)^Y-(1+r)^Y}{(1+r)^Y}\right)}{S \times a^{(i-e)}_{\overline{Y|}}}$$

Where

f-pension accrued per year of service

Y-number of years of service to be included in the caculation

S-current salary

e-inflationary revaluation(other than salary growth) of the pension before payment starts

r-inflationary revaluation(due to salary growth) of the pension before payment starts

i, v, r_{65}, l_x, AL are the standard actuarial and pension terms

$a_{\overline{Y|}}$ is an annuity payable for a limited period which is subject to all of the in-force decrements applicable for active members at the ages passed through.

State the value (or nature) of r and Y under the following methods

(i) Entry Age (EA) Method and the Attained Age Method. [**5 Marks**]

(ii) Basic form of Projected Unit (PU) Method and the Current Unit Method. [**5 Marks**]

(iii) Revaluation adjusted (CU) Method. [**3 Marks**]

QUESTION 4[20 MARKS]

(a) Describe what is meant by is a notional portfolio of assets.   [**7 Marks**]

(b) Consider a government fixed interest investment. Show that, for this investment,

$$\text{Discounted Value} = MV_{FI} \times (gry \times a_{\overline{n}} + v^n)@i\%$$

Where gry- gross redemption yield for the index n-the term of the index

[**8 Marks**]

(c) A United Kingdom pension fund has 3 pounds of its equity assets invested in various trusts. A broad discounted income model is used to value the assets and is assumed that the FTSE Actuaries Share Indices Investment Trusts. The dividends yield on this index at the valuation date is 3.40%. If the valuation rate of interest is 9% p.a and the assured rate of dividend, g, is 5% p.a . Find the discounted value of these assets at valuation date. [**5 Marks**]

QUESTION 5[20 MARKS]

The projected benefit outgo in a future year t can be expressed as

$$\sum_{k=1}^{N_t} B_{k,t}$$

where is the benefits paid to the beneficially in year t and N_t is the total number of beneficiaries in year t.

(a) (i) Suppose a level pension of amount $B(k,t)$is currently payable to a closed group of N individuals aged x last birthday.Find, basing on the above expression, the expression for the pension outgo in the future year t, given that $_tp_x$ is the probability that a life aged x survives to age x+t.

[**4 Marks**]

(ii) List at least four main features that may be incorporated in the expression above to have a more realistic model   [**4 Marks**]

(b) Describe, using relevant algebraic expressions, how the model above may be adjusted to have each of the following valuation models

(i) Emerging Cash Flow model.   [**4 Marks**]

(ii) Benefit Event Model.   [**4 Marks**]

(iii) Commutation Functions Model   [**4 Marks**]