JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

YEAR TWO SEMESTER ONE EXAMINATIONS

SMA 210: PROBABILITY AND DISTRIBUTION THEORY 1

AUGUST 2013

BED. SCIENCE, BED. ARTS AND SPECIAL NEEDS EDUCATION

TIME 2 HOURS

INSTRUCTIONS:

1. Answer question ONE(compulsory) and ANY other TWO questions in this paper
2. Answer all the questions in the answer booklet provided.
3. Do not write on this paper.

QUESTION ONE (COMPULSORY) – (30 MARKS)

a) The joint probability function for the random variables X and Y is tabulated as shown

	Y=0	Y=1	Y=2
X=0	0.03	0.08	0.06
X=1	0.08	0.07	0.04
X=2	0.04	0.15	0.22
X=3	0.07	0.12	0.04

Determine:

i. The marginal probability functions of X and Y

ii. P(0≤X≤2,Y<2)

(4marks)

b) Find the third raw moment for the exponential distribution with parameter Θ given as follows

$$f(x,\theta)=\begin{cases}\frac{1}{\theta}e^{-x/\theta}, x>0,\theta>0\\ 0, otherwise\end{cases}$$ (4marks)

c) The joint probability function of two discrete random variables X and Y is given by $f(x,y)=\begin{cases}k(2x-y),\ 0\le x\le 2, 0\le y\le 3\\ 0,\ otherwise\end{cases}$

Obtain the value of k. Are X and Y independent?(6marks)

d) Evaluate $\frac{\Gamma(9+4)\Gamma(4.5)}{\Gamma 6\Gamma 8}$ (5marks)

e) The joint p.d.f of three random variables X , Y and Z is defined as follows

$$f(x,y)=\begin{cases}c(xy+z), & 0<x<2, 0<y<4, 0<z<1\\ 0, & otherwise\end{cases}$$

Calculatethe value of c hence the conditional distribution of X given Y=y, Z=z

(7marks)

f) If X and Y have the joint density function $f(x)=\begin{cases}6x(1-x), 0<x<1\\ 0,\ elsew\ \ ere\end{cases}$, find the probability density of $Y=16X^2$ (4marks)

QUESTION TWO (20 MARKS)

a) Show that the moment generating function for the gamma distribution

$$f(x) = \begin{cases} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}, & x > 0, \alpha > 0, \beta > 0 \\ 0, & x \leq 0 \end{cases}$$,with scale parameter β

and shape parameter α is given by $M_x t = (1 - \beta t)^{-\alpha}$ hence or otherwise find the moment generating function for a random variable X which is exponentially distributed with parameter θ. (8marks)

b) Determine the value of c for which the function below is a joint probability density function hence give the marginal distribution of X.

$$f(x,y) = \begin{cases} c(x+y), & 0 < x < 3, x < y < x+2 \\ 0, & otherwise \end{cases}$$

(6marks)

c) Show that the fifth raw moment for the uniform distribution with parameters a and $b, b > a$ is $1/_6\,(b^3 + a^3)(b^2 + ab + a^2$ (6marks)

QUESTION THREE (20 MARKS)

a) Given $f(x,y) = \begin{cases} \frac{1}{9}(xy), & 0 < x < 2, 0 < y < 3 \\ 0, & otherwise \end{cases}$ obtain the variance covariance matrix. Are X and Y independent? (10marks)

b) Compute the third raw moment for a random variable X at $\alpha = 8, \beta = 14$, given that X assumes the distribution $f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}, & 0 < x < 1, \alpha > 0, \beta > 0 \\ 0, & otherwise \end{cases}$

(6marks)

c) X and Y have the bivariate normal distribution with parameters stated as follows: $\underline{\mu} = \binom{3}{7}$,$\Sigma = \begin{bmatrix} 9 & 2.4 \\ 2.4 & 4 \end{bmatrix}$. Compute $E(X/Y = 8.5)$ and $var\,(X/Y = y)$ (4marks)

QUESTION FOUR (20 MARKS)

a) Suppose X and Y are two discrete random variables whose joint p.m.f is tabulated as follows

	Y=0	Y=1	Y=2	f(x)
X=0	0.10	0.10	0.20	0.40
X=1	0.0	0.15	0.05	0.20
X=3	0.10	0.20	0.10	0.40
f(y)	0.20	0.45	0.35	1

Use the table to compute $cov(X,Y)$. (6marks)

b) Given $f(x) = \begin{cases} \frac{3}{4}(2x - x^2), 0 < x < 2 \\ 0, otherwise \end{cases}$ obtain the distribution for a new random variable $V = 4X^2$ (4marks)

c) Let $f(x) = \begin{cases} abx^{b-1}e^{-ax^b}, x > 0 \\ 0, \ otherwise \end{cases}$

Obtain a general expression for the mean and variance of X (10marks)

QUESTION FIVE (20 MARKS)

a) Let X and Y be two independent standard normal random variables. Let $U = X - Y$ and $V = X + Y$ be two new random variables. Determine the joint p.d.f of U and V.

(8marks)

b) Let $f(x,y) = \begin{cases} \frac{1}{30}(x+y), \ x = 0,1,2 \, ; y = 0,1,2,3 \\ 0, \ otherwise \end{cases}$. Show that $E(X - Y)$=$E(X) - E(Y)$

(8marks)

c) Let X and Y have the joint density function $f(x,y) = \begin{cases} 2e^{-x-2y}, \ x > 0 \, , y > 0 \\ 0, \ otherwise \end{cases}$

Check whether or not X and Y independent? (4marks)