JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE & TECHNOLOGY UNIVERSITY EXAMINATIONS 2012/2013

2ND YEAR 1ST SEMESTER EXAMINATION OF BACHELOR OF EDUCATION (SCIENCE)

REGULAR

COURSE CODE: SMA 301

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS I

DATE: 21/8/13 **TIME: 2.00 -4.00 PM**

DURATION: 2 HOURS

INSTRUCTIONS

1. This paper contains five (5) questions.
2. Answer question 1 (compulsory) and ANY other TWO questions.
3. Write all answer in the booklet provided.

SMA 301 : ORDINARY DIFFERENTIAL EQUATION I

Attempt question 1 and any other two questions

QUESTION 1 COMPULSORY(30 marks)

a) Consider $y = A\sin x - B\cos x$, where A and B are arbitrary constants. By eliminating the arbitrary constants through differentiation form a differential equation. State its degree and order. (3 marks)

b) The rate of decay of a radioactive material is given by $\frac{dN}{dt} = -ªN$ where ª , is a decay constant and $ªN$ the number of radioactive atoms disintegrating per second. Determine the half life in years of a nickel isotope assuming the decay constant is 1.832×10^{-10} atoms per second and a 365 day year.(Half life means the time taken for N to become one half of the original) (4 marks)

c) Reduce the differential equation $(y - x - 2)dy = (y + x - 6)dx$ to homogeneous form hence solve. (6 marks)

d) Solve the differential equation $xy'' = y' + (y')^3$ (6 marks)

e) Find the particular solution to the differential equation

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that x=0, y = 1 and $\frac{dy}{dx} = -2$ (6 marks)

f) Use variation of parameters to solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}$ (5 marks)

QUESTION 2(20 MARKS)

Use any appropriate method to solve the differential equations below

a) $(1 - x^2)\frac{dy}{dx} + xy = x(1 - x^2)\sqrt{y}$ (8 marks)

b) $(\sqrt{xy} - x)dy + ydx = 0$ (8 marks)

c) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$ (6 marks)

QUESTION 3(20 MARKS)

Find the general solution of each of the following differential equation

a) $yy'' + (y')^2 = 0$ (6 marks)

b) $(1 + x^2)dy = (\tan^{-1} x - y)dx$ (8 marks)

c) $\left(x\tan\frac{y}{x} - y\sec^2\frac{y}{x} \right)dx + x\sec^2\frac{y}{x}dy = 0$ (6 marks)

QUESTION 4(20 MARKS)

a) Find the general solution of

(i) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$ (4 marks)

(ii) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ (4 marks)

(iii) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 10y = 0$ (4 marks)

b) Use the method of variation of parameters to find the solution of

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 10$ (8 marks)

QUESTION 5(20 MARKS)

a) Solve the differential equation $2yy'' = (y')^2 + 1$ (5 marks)

In an L-C-R circuit, the charge q on a plate of condenser is given by $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E\sin\mu t$ where $i = \frac{dq}{dt}$. The circuit is tuned to resonance so that $\mu^2 = \frac{1}{LC}$. If $R^2 < \frac{4L}{C}$ and $q = 0 = i$ when $t = 0$.

Show that $q = \frac{E}{R\mu}\left\{-\cos\mu t + e^{\frac{-Rt}{2L}}\left(\cos pt + \frac{R}{2LP}\sin pt\right)\right\}$

and $i = \frac{E}{R}\left(\sin\mu t - \frac{1}{P\sqrt{LC}}e^{-\frac{Rt}{2L}}\sin Pt\right)$

where $P^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$ (15 marks)