JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (ACTUARIAL) WITH IT

1ST YEAR 1ST SEMESTER 2013/2014 ACADEMIC YEAR

CENTRE: MAIN

COURSE CODE: SAS 103

COURSE TITLE: INTRODUCTION TO PROBABILITY THEORY

EXAM VENUE: LR 1 **STREAM: (Actuarial)**

DATE: 23/12/2013 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2 HOURS

Instructions:

1. **Answer question 1 (compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)-(COMPULSORY)

a. In the probability distribution below it is known that $E(X) = 3.2$

X	**1**	**2**	**3**	**4**	**5**
Pr(X=x)	**0.3**	**a**	**0.1**	**0.3**	**b**

i. Compute the values of a and b. (3 marks)

ii. Determine $Var(2X)$ (5 Marks)

b. A continuous random variable X has the p.d.f given by

$$f(x) = \begin{cases} K(2x - x^2), & 0 < x < 2 \\ 0, & otherwise \end{cases}$$

Find

i. the value of the constant K (3marks)

ii. $P(X < {}^{1}/_{K})$ (3marks)

c. The number of surface flaws in plastic panel used in the interior of automobiles has a Poisson distribution with a mean of 0.06 flaws per square foot of plastic panel. Assume each automobile interior contains 20 square feet of plastic panel. Find the probability that:

i. There are no surface flaws in an auto's interior. (3marks)

ii. In 10 cars sold to a rental company none of the 10 cars has any surface flaw. (2marks)

iii. In 10 cars sold to a rental company at most two cars have any surface flaw. (4marks)

d. Two cards are selected from a box which contains seven cards numbered 1,1,2,2, 3 ,6 and 8. Let X denote the sum of the pair drawn. Find

i. the distributions of X. (4marks)

ii. the probability of getting a prime sum. (3marks)

QUESTION TWO (20 MARKS)

a. The comprehensive strength of samples of cement can be modeled by a normal distribution with mean of 6000 kilograms per square centimeter and a standard deviation of 100 kilograms per square centimeter. Find:

i. The probability that a sample's strength is less than 6250 kg/cm^2

ii. The probability that a sample's strength is between 5800kg/ cm^{2}and 5900kg/ cm^2

iii. The strength exceeded by 95% of the samples. (10 marks)

b. The binomial distribution $b(x, n, \theta)$ is known to have the mean $n\theta$. Derive for this distribution $E(X^2)$ hence$var\ (x)$. Obtain the standard deviation of x at $n = 50$,
$1 - \theta = 0.38$. (10 marks)

QUESTION THREE (20 MARKS)

a. Determine the p.m.f, the second raw moment and the moment generating function at t=1 of a discrete random variable with the following c.d.f. (8marks)

F(x)=	0	$x < 2$
	0.2	$2 \leq x < 5.7$
	0.5	$5.7 \leq x < 6.5$
	0.8	$6.5 \leq x < 8.5$
	1	$8.5 \leq x$

b. The negative binomial distribution for a random variable X is given as follows

$$f(x) = \binom{x-1}{k-1} \alpha^k (1-\alpha)^{x-k}$$

,

k=1, 2, 3…

x=k, k+1, k+2…

Derive expressions for E(X) and Var (X) .

Hence find the standard deviation given

k=4, α=0.02 (12 marks)

QUESTION FOUR (20 MARKS)

a. Verify that $f(x) = \begin{cases} \left(\frac{512}{27}\right)^{1/3} \left(\frac{3}{11}\right)^x, & x = 1,2,3,4, \ldots. \\ 0, & otherwise \end{cases}$

is a p.m.f hence compute $P(X > 2)$

(7marks)

b. A pair of digits from 1 to 4 is chosen at random with repetitions allowed. Let X denote the product of the digits. Find the distribution of X hence its variance. (7 marks)

c. The probability of a successful optical alignment in the assembly of an optical data storage product is 0.8. Assume that the trials are independent. What is the probability that:

   i. The first successful alignment requires four trials.
   ii. The first successful alignment requires at least four trials. (6 marks)

QUESTION FIVE (20 MARKS)

a. Let A and B be events with $P(A) = 0.3$, $P(B) = q$, $P(AUB) = 0.5$. Find q if

   i. A and B are mutually exclusive.
   ii. A and B are Independent (5 marks)

b. During a school vacation, I can go skiing, hiking or stay at home and play soccer with the odds 50%, 30% and 20% respectively. The conditional probabilities of getting injured are 30%, 10% and 20% respectively if I go skiing, hiking or play soccer. Find the probability that:

   i. I will get injured.
   ii. If I came back from vacation with an injury, I had gone skiing.

(7 marks)

c. The chance of John passing a standard examination is 0.6. Let X be a random variable representing the number of passes in 5 standard examinations.

   i. Using an appropriate assumption, find the probability distribution of X. You may round off probabilities to 3 decimal places.
   ii. If for every standard examination passed he earns a token of shs. 2,000 find the expected amount of money after the 5^{th} pass.

(8 marks)