JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (ACTUARIAL) WITH IT

1ST YEAR 2ND SEMESTER 2013/2014 ACADEMIC YEAR

CENTRE: KISII L. CENTRE FULL TIME

COURSE CODE: SMA 102

COURSE TITLE: CALCULUS I

EXAM VENUE: LR 17 **STREAM: (BSc. Actuarial)**

DATE: 23/4/2014 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2 HOURS

Instructions:

1. **Answer question 1 (compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Use the ϵ-δ definition to show that $\lim_{x\to 2} x^2 = 4$ (4mks)

b) Evaluate the following limits

i) $\lim_{x\to -3} \frac{x+3}{x^2+7x+12}$ (3mks)

ii) $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$ (4mks)

c) i) Define the term continuity of a function *f* (1mk)

ii) Let $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 1, & x = 3 \end{cases}$

Find whether this function is continuous or not at $x=3$? (3 mks)

d) Find $\frac{dy}{dx}$ using implicit differentiation if $x^5 + 4xy^3 - 3y^5 = 2$ (5mks)

e) Find the tangent and normal linesto the curve $y^2 - 6x^2 = -4y - 19$ at the point (2,1) (5mks)

f) Investigate the stationary values of the function $y = x^4 - 4x^3$ (5mks)

QUESTION TWO (20 MARKS)

a) The parametric equation of a curve are as follows: $x = e^t$, $y = \sin t$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of *t*, hence show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (6mks)

b) i) A rectangular sheet of metal having dimensions *20cm by 12cm* has squares removed from each of the four corners and the sides bent upwards to form an open box. Determine the maximum possible volume of the box (6mks)

ii) Find the value of the constant *k* if the line $y - 3x + k = 0$ is tangent to the curve $y = 2x^2$. Also find the point of tangency (4mks)

c) i) State the intermediate value theorem (1mk)

ii) Show that the function $f(x) = x^3 - 2x + 1$ defined on $[-3, 2]$ has a real root (3mks)

QUESTION THREE (20MARKS)

a) Find the equation of the tangent line to the parabola $y=x^2$ at the point $(1,1)$ (4mks)

b) A curve is represented parametrically by $x=\sqrt{t}$ and $y=t-\frac{1}{\sqrt{t}}$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t (6mks)

c) A particle moves from a point A so that after t seconds it is 5 metres from A where $S=8t-t^2$ .Find the velocity when (4mks)

i) $t=0$ seconds

ii) $t=4$ seconds

d) Evaluate the following limits

iii) $\lim_{x\to 0} \frac{\sqrt{x+2}-9}{x}$ (3mks)

iv) $\lim_{x\to \infty} \frac{x+\sin x-9}{2x+5}$ (3mks)

QUESTION FOUR (20 MARKS)

a) Find $\frac{dy}{dx}$ given that

i) $y=e^{2x+\sin x}$ (4mks)

ii) $xy+x-2y+1=0$ (4mks)

b) The motion of a student's bicycle can be expressed by the distance equation $S=t^3+2t$. Find the acceleration function and hence determine the acceleration at $t=4s$.(5mks)

c) Find the value of y'' at the point (-1,1) of the function $x^2y+3y-4=0$ (7mks)

QUESTION FIVE (20 MARKS)

a) Use logarithmic differentiation to find the derivatives of

i) $y=\ln(x^2+3x-5)$ (3mks)

ii) $y=\ln\left(\frac{4x-1}{x+1}\right)^{\frac{1}{2}}$ (3mks)

b) Use first principles to find the derivative of $y=x+\frac{1}{x}$; $x\neq 0$ (6mks)

c) A 2% error is made in measuring the radius of a sphere. Find the percentage error in its surface area. (4mks)

d) Find the turning points of the curve $y=5+24x-9x^2-2x^3$ and distinguish them

(4mks)