



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL

2ND YEAR 1ST SEMESTER 2015/2016 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 203

COURSE TITLE: FUNDAMENTALS OF ACTUARIAL SCIENCE 2

EXAM VENUE: LAB 5

STREAM: BSc. Actuarial Sc.

DATE: 22/04/16

EXAM SESSION: 9.00 – 11.00 am

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE [30 MARKS]

1a). Given ${}_tP_{30} = \frac{7800 - 70t - t^2}{7800}$ for $0 \leq t \leq 100 - x$

i) Find ${}_{10}P_{30}$ and ${}_1P_{30}$ and hence find ${}_9P_{31}$ [4 marks]

ii) Find μ_{30+t} and then μ_{30} and μ_{40} [4 marks]

iii) Calculate ${}_{10}P_{31}$ assuming μ_{40} to be constant between ages 40 and 41 [2 marks]

b) Given ${}_tP_x = \left[\frac{1+x}{1+x+t} \right]^3$ for $t \geq 0$

Calculate the complete life expectancy of a person aged $x = 41$ [5 marks]

2). A single premium life insurance policy with benefits payable at the end of the year of death is issued to (x). Suppose that :

- $A_x = 0.25$; $d = 0.05$
- Sales commission is 18% of gross premium ; Taxes are 2% of gross premium
- Per policy expenses are \$ 40 the first year and \$ 5 per year after

Calculate the policy premium that should be charged [6 marks]

3) A life office issued a 20 year endowment assurance to a person aged 25. The sum assured is \$10,000 payable immediately of death or on survival for 20 years, Using A1967-70 ultimate 6 % and ignoring expenses, Calculate the standard deviation of the benefit [7 marks]

4) Define :

- surrender value [2 marks]
- Reserve [2 marks]

QUESTION 2[20 marks]

1) You are given the following: [6 marks]

$$i = 3\%$$

$$A_{30} = 0.27921$$

$$\ddot{a}_{40} = 21.68$$

$${}_{10}E_{40} = 0.73756$$

Estimate the value of ${}_{10}P^{(12)}(\bar{A}_{30})$ and interpret this actuarial symbol

2). An office issues a large number of 25 year endowment assurance to lives aged exactly 40. Level annual premiums are payable throughout the term and the sum assured of each policy is \$ 10,000 payable at the end of year of death or on survival to the end of the term. The office premium basis is:

Mortality: A1967-70 ultimate

Rate: 4%

Expenses: 5% of each annual premium including the first with additional expenses of 1% of the sum assured

[6 marks]

3.a) Find the single premium for a 10 year term assurance with the sum assured for \$ 500 for a life aged 30 on the basis of the 1967-70 select table at 4%. The premium is paid to be returned with the sum insured in the event of a claim. Expenses are 3% of the sum assured with continuing administrative costs of \$ 1 each year [6 marks]

b). State the conditions under which the prospective and the retrospective reserves will be equal [2 marks]

QUESTION THREE [20 MARKS]

1) Ten years ago a life office issued a 30 year endowment assurance without profits to a person aged 35. The sum assured is \$10,000 payable at the end of year of death or on survival for 30 years, and premiums are payable annually in advance. Using A1967-70 ultimate 5% and ignoring expenses, Calculate

- i. The annual premium [4 marks]
- ii. The reserve, assuming that premium now due has been paid [3 marks]

2). A life aged exactly 20 buys a 15 year endowment assurance policy with a sum assured of kshs. 100,000 payable on maturity or at the end of year of earlier death. Level of premiums are payable monthly in advance. Calculating the monthly premium, assuming A1967-70 ultimate mortality at 6% interest. Ignore the expenses. [5 marks]

3) Evaluate the following functions assuming the given basis: (A1967-70 ultimate mortality and interest rate of 6% pa) [4 marks]

a) $\ddot{a}_{65:20}$ b) $A_{68:2}$

4.a). Calculate by means of commutation functions on the basis of A1976-70 tables at 4% interest the value of: [2 marks]

i. $A_{[45]:15}$ ii. ${}_5\ddot{a}_{45}$

b). Calculate the premium values using the of A1967-70 table at 6% interest [2 marks]

i. $P_{[40]:10}$ ii. ${}_5P_{40}$

QUESTION FOUR [20 MARKS]

1. A mortality table is defined such that

$${}_t p_x = \left(1 - \frac{t}{110-x}\right)^{\frac{1}{2}} \quad \text{for } x < 110, t < 110-x$$

And ${}_t p_x = 0$, for $t \geq 110-x$

Calculate:

i. The complete expectation of life at exact age 45 [4 marks]

ii. The force of mortality at age 45 [3 marks]

2..For a certain animal population

$$l_x = \frac{l_0}{(1+x)^2} \quad (x \geq 0)$$

Calculate

i. The complete expectation of life at birth [2 marks]

ii. The force of mortality at age 1 year [2 marks]

iii. The chance that a newly born animal will die between ages 1 and 2 years [1 mark]

3) In a certain population, μ_x is given by

Interval	μ_x
$60 \leq x \leq 70$	0.01
$70 \leq x \leq 80$	0.015
$x > 80$	0.025

Calculate the probability that a life aged exactly 65 will die between ages 80 and 83 [3 marks]

4.) Suppose that $s(x) = (1 - x/\omega)^\alpha$

Give simple formulae for

[5 marks]

i. μ_x
[1 mark]

ii. e_x^0
[2 marks]

iii. ${}_{10}p_{70}$
[1 mark]

- iv. ${}_{40/5}q_{35}$
[1 mark]

QUESTION FIVE [20 MARKS]

1). For many years a company has recruited ,uniformly over each year,200 employees on their 20th birthdays and a fixed number of additional employees on their 25th birthdays. Mortality follows English Life table no.-12 males. Employees may retire on their 60th or 65th birthdays, and one third of employees reaching their 60th birthdays retire on that date. Employees leave the company only through death or retirement and the total number of employees is 10,000.Find the total number of new recruits each year [5 marks]

2).A 20 year temporary annuity due of 1 per annum is issued to a life aged 50 exact. [4 marks]

a)Express the expected present value of the annuity in terms of an assurance function

b)Hence calculate the value using the mortality table A1967-70 ultimate with 4% interest

3).A country's school system provides education for all children between ages 5 and 16 exactly.The country's population is stationary there being 100,000 births per year uniformly distributed over the year.The population of the country is subject to the mortality of ELT No.12-males . [7 marks]

i.Find the number of pupils at any given time

ii.The country's teacher training colleges are such that a constant flow of new entrants to the profession is maintained . Teachers are recruited uniformly over the year,and the ratio of pupils to teachers is 20 to 1.All teachers enter the profession at age 21 and retire at age 60, there being no withdrawals.Find the annual number of teachers recruited.

4).Using English life table no.12 –male calculate

i. $e_{2:10}^0$
[2 marks]

ii. T_{15}
[2 marks]