JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

1ST YEAR 1ST SEMESTER 2015/2016 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 103

COURSE TITLE: MATHEMATICAL MODELING

EXAM VENUE: LAB 2 **STREAM: (BSc. Actuarial)**

DATE: 21/04/16 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

a) Differentiate between difference equation and differential equation
(2marks)

b) Find the first four terms of the solution to

$$x(t+1) - x(t) = -\frac{[x(t)]^2 - 4x(t) + 4}{x(t)}$$

which satisfies the initial condition $x(0) = 4$.
(5marks)

c) Suppose that the supply and demand equations are given by $D(n) = -2p(n) + 3$ and $S(n+1) = p^2(n) + 1$.

(a) Assuming that the market price is the price at which supply equals demand, find a difference equation that relates $p(n+1)$ to $p(n)$.

(b) Find the positive equilibrium value of this equation.
(5marks)

d) Solve the following difference equation hence find x_8
(5marks)

$$x_{n+1} - x_n = -0.3x_n + 2, \quad \text{with } x_0 = 7.$$

e) Solve this differential equation.

$$\frac{dN}{dt} = k(65 - N), \quad 0 \le N \le 65$$

(5marks)

f) An employee joined a company in 2009 with a starting salary of $50,000. Every year this employee receives a raise of $1000 plus 5% of the salary of the previous year.

a) Set up a difference equation for the salary of this employee *n* years after 2009.

b) What will the salary of this employee be in 2017?
(6marks)

g) A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria. Find the number of bacteria after 4 hours.
(6marks)

QUESTION TWO (20 marks)

a) Suppose that you can get a 30-year mortgage at 8% interest p.a. How much can you afford to borrow if you can afford to make a monthly payment of $1,000?
(10marks)

b) A new drug is introduced through an advertising campaign to a population of 1 million potential customers. The rate at which the population hears about the drug is assumed to be proportional to the number of people who are not yet aware of the drug. By the end of 1 year, half of the population has heard of the drug. How many will have heard of it by the end of 3 years?
(10marks)

QUESTION THREE (20 marks)

a) A debt of $12,000 is to be amortized by equal payments of $380 at the end of each month, plus a final partial payment one month after the last $380 is paid. If interest is at an annual rate of 12% compounded monthly, construct an amortization schedule to show the required payments.
(12marks)

b) Find a solution for the equation

$$x(n+1) = 2x(n) + 3^n, \qquad x(1) = 0.5.$$

(8marks)

QUESTION FOUR (20 marks)

You have a balance of $5,000 on a credit card which charges 2% interest per month. You promise to pay p dollars a month to the credit card company and not make any new charges.

(a) Formulate a model in terms of p which allows you to pay off the credit card in 10 years.
(b) Solve your model analytically to find the value of p (to the nearest cent) which will allow you to pay off the credit card in exactly 10 years.

A population of weasels has a natural growth rate of 3% per year. Let w_n be the number of weasels n years from now and suppose there are currently 300 weasels.
(a) Suppose the carrying capacity of the weasel's habitat is 1000. Using an inhibited growth model, write a difference equation which describes how the population changes from year to year.
(b) Using the difference equation from part (a), compute w_n for n = 1, 2, . . . , 10. (10marks)

QUESTION FIVE (20 marks)

Consider the supply and demand model below in the following cases:

$$P_n = aP_{n-1} + b$$

where $a = -m_s / m_d$ and $b = (c_d - c_s) / m_d$.

Unit increase in price, produces an increase of m_s units in supply, m_s represents the *sensitivity of suppliers to price*. A one unit increase in price produces m_d units decrease in demand, m_d represents the *sensitivity of consumers to price*, c_s and c_d *are constants*

Case 1: $m_s = 0.25,\ m_d = 0.5,\ c_s = 2,\ \mathrm{c_d} = 8,$ and $P_0 = 10$

Case 3: $m_s = 0.6,\ m_d = 0.5,\ c_s = 1,\ \mathrm{c_d} = 7.6,$ and $P_0 = 6.5$

For each case do the following:

(i) Find the equilibrium price P_e . Find a numerical solution (n, P_n); 0,1,...,14 , graph it and analyze its behavior.

(ii) Determine whether the equilibrium price P_e is stable.

(20marks)