



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE**

**ACTUARIAL**

**3<sup>RD</sup> YEAR 1<sup>ST</sup> SEMESTER 2015/2016 ACADEMIC YEAR**

**MAIN REGULAR**

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**COURSE CODE: SAC 305**

**COURSE TITLE: PENSION MATHEMATICS**

**EXAM VENUE:**

**STREAM: (BSc. Actuarial)**

**DATE:**

**EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION 1 [COMPULSORY] [30 Marks]**

(a) State giving the features, the types of pension schemes [6 Marks]

(b) The valuation assumptions for a pension scheme are  $i = 9\%$ , and  $e = 7\%$ . What is the percentage change in the value of liabilities, with a mean term to retirement of 15years if:

(i)  $i$  and  $e$  are both reduced by 1%p.a? [2 Marks]

(ii)  $i$  and  $e$  are both increased by 3%p.a? [2 Marks]

(iii)  $i$  is unchanged but  $e$  is reduced by 1%? [2 Marks]

(c)(i) What is the main factor that affects the risk that a pension guarantee is not met? [3 Marks]

(ii) Give three examples of options that may be given to an individual in regard to pension scheme benefits. [3 Marks]

(d) A portfolio might consist of:

Domestic equity 55%

Overseas equity 20%

Property 3%

Index-linked bonds 7%

Fixed interest bonds 15%

State two possible homogenous grouping of the above assets. [4 Marks]

(e) The value of a pension P payable on ill-health retirement to an individual currently aged x could be expressed as

$$P \sum_{y=x}^{64} v^{y+\frac{1}{2}-x} \frac{i_y}{l_x} a_{y+\frac{1}{2}}^i = \frac{P}{D_x} \sum_{t=0}^{64-x} C_{x+t}^{ia}$$

$$= P \frac{M_x^{ia}}{D_x}$$

(i) Define  $C_{x+t}^{ia}$  and  $M_x^{ia}$  as used above. [2 Marks]

(ii) Given

$$M_{50}^{ia} = 1421$$

$$M_{50}^{ra} = 16472$$

$$D_{50} = 3222$$

Calculate the present value of an annuity of Kshs.1,000,000 p.a payable on retirement at any age up to 65, due to;

(a) ill-health retirement. [3 Marks]

(b) normal retirement [3 Marks]

Assume the member is currently aged 50.

## QUESTION 2[20 MARKS]

(a) Using the following assumptions and data  $i = 10\%$ ,  $e = 8\%$ ,  $A = 60$ ,  $R = 65$  and  $a'_R = 12$ , calculate the AL and SCR under the Projected Unit for the following members individually;

(i) 25 year old, no past service, salary Kshs.20,000 [4 Marks]

(ii) 40 year old, 15 years past service, salary Kshs.15,000 [4 Marks]

(iii) 55 year old, 30 years past service, salary Kshs.30,000 [4 Marks]

The same earnings definition is used for benefit purposes. Assume for simplicity, that contributions are paid continuously and salary growth is continuous.

(b) You are the actuary to a pension scheme which has only two members. One of the members is aged 20 and has recently joined the scheme with a pensionable salary of Kshs.10,000. The other member is aged 59 and has a pensionable salary of Kshs.100,000.

You have been asked to recommend a contribution rate for this scheme. Currently the scheme has no surplus or deficit if the PU method is used. Calculate the PUSCR and AASCR, for this data using the following assumption data and comment on your answer.

$$i = 9\%$$

$$e = 7\%$$

$$A = 60\%(\text{ie accrual rate is 60ths})$$

$$R = 60$$

$$a'_R = 15$$

[8 Marks]

### QUESTION 3[20 MARKS]

You are the actuary to a long established defined benefit pension scheme that was closed to new entrants after the previous valuation. The next

actuarial valuation is due.

(i) Discuss the characteristics of the Projected Unit and Attained Age funding methods. [8 Marks]

(ii) Set out simple formulae (ignoring any pre-retirement decrements) that could be used to calculate the Standard Contribution Rates for the Projected Unit and Attained Age funding methods respectively. [5 Marks]

(iii) Explain how you could estimate the Standard Contribution Rate for the Attained Age funding method from the calculated Standard Contribution Rate for the Projected Unit funding method. [2 Marks]

(iv) Set out the different ways company contributions could be structured to eliminate a funding deficit and outline the characteristics of each method. [5 Marks]

#### **QUESTION 4[20 MARKS]**

(a) A pension scheme provides a pension of  $\frac{1}{60}$  of career average salary in respect of each full year of service, on age retirement between the ages of 60 and 65. A proportionate amount is provided in respect of an incomplete year of service. At the valuation date of the scheme, a new member aged exactly 40 has an annual rate of salary of Kshs.40,000.

Calculate the expected present value of the future service pension on age retirement in respect of this member, using the Pension Fund Tables in the Formulae and Tables for Actuarial Examinations. [5 Marks]

(b) A member of a pension scheme is aged exactly 40, having joined the scheme at age exactly 22. He earned Kshs.30,000 in the immediately preceding 12 months. Final pensionable salary is defined as the annual average earnings over the three years immediately prior to retirement. Normal Retirement Age is a member's 65th birthday.

Using the functions and symbols defined in, and assumptions underlying, the Example Pension Scheme Table in the Actuarial Tables, calculate the expected present value of each of the following:

(i) A pension on ill-health retirement of two-thirds of final pensionable salary. **[5 Marks]**

(ii) A pension on retirement at any stage on grounds other than ill-health of one eightieth of final pensionable salary for each year of service (fractions of a year counting proportionately), subject to a maximum of 40 years.

**[5 Marks]**

(iii) A lump sum on retirement at any age for any reason of 0,000.

**[5 Marks]**

### **QUESTION 5[20 MARKS]**

The projected benefit outgo in a future year  $t$  can be expressed as

$$\sum_{k=1}^{N_t} B_{k,t}$$

where is the benefits paid to the beneficiary in year  $t$  and  $N_t$  is the total number of beneficiaries in year  $t$ .

(a) (i) Suppose a level pension of amount  $B(k, t)$  is currently payable to a closed group of  $N$  individuals aged  $x$  last birthday. Find, basing on the above expression, the expression for the pension outgo in the future year  $t$ , given that  ${}_t p_x$  is the probability that a life aged  $x$  survives to age  $x+t$ .

[4 Marks]

(ii) List at least four main features that may be incorporated in the expression above to have a more realistic model

[4 Marks]

(b) Describe, using relevant algebraic expressions, how the model above may be adjusted to have each of the following valuation models

(i) Emerging Cash Flow model.

[4 Marks]

(ii) Benefit Event Model.

[4 Marks]

(iii) Commutation Functions Model

[4 Marks]