

JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND  
TECHNOLOGY

4<sup>th</sup> YEAR 1<sup>st</sup> SEMESTER EXAMINATION [SCHOOL BASED]

SMA 403: TOPOLOGY

**INSTRUCTION:** Attempt question one (COMPULSORY) and any other TWO questions only.

**QUESTION ONE (COMPULSORY) [30 MARKS]**

- (a). Given the set  $W = \{w : w \text{ is a consonant}\}$ , find the cardinality of  $W$  and its complement. (3 marks)
- (b). State and prove the generalized 1<sup>st</sup> De' Morgan's Law. (5 marks)
- (c). Distinguish between a regular space and a basis. (4 marks)
- (d). Show that the constant function is continuous. (4 marks)
- (e). Show that a set is closed if its complement is open. (4 marks)
- (f). Let  $X$  be a topological space, and let  $A$  be a subset of  $X$ . A subset  $B$  of  $A$  is closed in  $A$  if and only if  $B = A \setminus F$  for some closed subset  $F$  of  $X$ . (5 marks)
- (g). Define: Neighbourhood, closed set and accumulation point. (5 marks)

**QUESTION TWO [20 MARKS]**

- (a). Show that the set  $C_R[0, 1]$  of all real-valued continuous functions defined on  $[0, 1]$  is a metric space with respect to the metric defined as  $d(f, g) = \sup\{|f(x) - g(x)|; x \in [0, 1]\}$  where  $f, g \in C_R[0, 1]$ . (10 marks)
- (b). Show that  $(X, \mathfrak{D})$  is a topological space where  $\mathfrak{D}$  is the class of all subsets of  $X$ . (10 marks)

**QUESTION THREE [20 MARKS]**

- (a). State and prove the cardinality theorem for any three finite sets  $A$ ,  $B$  and  $C$ . (5 marks)
- (b). Prove that all metric spaces are  $T_2$  spaces. (15 marks)

**QUESTION FOUR [20 MARKS]**

- (a). Define a normal space. (2 marks)
- (b). If  $P = \{(a, b) \in \mathbb{R}^2 : ab = 1\}$  and  $Q = \{(a, b) \in \mathbb{R}^2 : b = 0\}$ , find  $d(P, Q)$ . (5 marks)
- (c). State and prove Cantor's Cardinality Theorem. (9 marks)
- (d). Describe two applications of the study of topology to real life situations. (4 marks)

**QUESTION FIVE [20 MARKS]**

- (a). Define continuity of a function between topological spaces (2 marks)
- (b). Let  $X, Y, Z$  be topological spaces, and let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be continuous functions. Then the composition  $g \circ f : X \rightarrow Z$  of the functions  $f$  and  $g$  is continuous. (14 marks)
- (c). Let  $X, Y$  be topological spaces, and let  $f : X \rightarrow Y$  be a function from  $X$  to  $Y$ . The function  $f$  is continuous if and only if  $f^{-1}(G)$  is closed in  $X$  for every closed subset  $G$  of  $Y$ . (4 marks)