JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FIRST YEAR FIRST SEMESTER EXAMINATIONS 2014

MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 818: ORDINARY DIFFERENTIAL EQUATIONS II

INSTRUCTION: Answer any THREE questions.

QUESTION ONE (20 MARKS)

a) Define Orthogonality (3 marks)

b)Given the Boundary value

$x^2 y'' + 2xy' + «y = 0$, $y(0) = 0$ and $y(e) = 0$

i) Show that it is a sturm – Liouville problem

ii) Find the eigen values and eigenfunctions

iii) Obtain the set of functions othornormal in the interval $1 \le x \le e$ (17 marks)

QUESTION TWO (20 MARKS)

Consider a Bessel Equation of order n given by $x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - n^2\right)y = 0$

a) By assuming a solution $y = a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + ...$ show that the roots of the indical equation are $m = n$ and $m = -n$. (3 marks)

b) From *a*) above use $m = n$ and $m = -n$ to obtain the possible bessel functions (3 marks)

c) Considering non integral and non zero values of *n* determine the complete solution of the Bessel's equation giving your answer in terms of Γ (gamma) (5 marks)

d) Taking $J_n(x) = \sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{r!\Gamma(n+r+1)}$ and leting the solution be $y = u(x)J_n(x)$ for integral values of *n* Show that the complete solution is $y = AJ_n(x) + BJ_n(x)\int \frac{dx}{x\left[J_n(x)\right]^2}$ (9 marks)

QUESTION THREE (20 MARKS)

Show that the indicial equation for $x(1-x)\frac{d^2 y}{dx^2} + (1-5x)\frac{dy}{dx} - 4y = 0$ has double root hence obtain the series solution given that $y(0) = 4, y'(0) = 5$

QUESTION FOUR (20 MARKS)

Use frobenius method to solve

$$\left(1-x^2\right)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

QUESTION FIVE (20 MARKS)

a) Determine the constants $\alpha_1, \alpha_2\ \alpha_3$, so that $f(x) = \alpha_1 x + 2$, α_1, $g(x) = \alpha_2 x^2 + \alpha_3 x + 1$ and $h(x) = x - 1$ are mutually orthogonal in $0 \le x \le 1$ and then obtain the corresponding orthonormal set (12 marks)

b) Solve the boundary value problem $y'' + 4y' + (4 + 9\alpha)y = 0$, $y(0) = 0$, $y(l) = 0$ (8 marks)