JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THEDEGREE OF BACHELOR OF EDUCATION (SCIENCE)

MAIN

REGULAR RESIT

COURSE CODE: SPH 203

COURSE TITLE: MATHEMATICAL METHODS FOR PHYSICS 1

EXAM VENUE: LAB 1 **STREAM: (BED SCI)**

DATE: 4/05/2016 **EXAM SESSION: 9:00-11:00AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer Question 1(compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

a. Using the differentiation from first principle, differentiate $f(x) = 4x^3 + 12x^2 - 4x + 1000$ (3 marks)

b. Use the Liebnitz theorem to evaluate the fourth derivative of the function $f(\theta) = \sin\theta\cos\theta$ (3 marks)

c. Evaluate $\int \sec x dx$ (4 marks)

d. Show how the convergence of the series $\sum_{n=r}^{\infty} \frac{(n-r)!}{n!}$ depends on the value of r (3 marks)

e. Find the sum, S_N, of the first N terms of the series, $\sum \ln\left(\frac{n+1}{n}\right)$ and hence determine whether the series is convergent, divergent or oscillatory. (4 marks)

f. Evaluate

$$\lim_{x\to 2} \frac{x^3 + x^2 - 5x - 2}{2x^3 - 7x^2 + 4x + 4}$$

(3 marks)

g. Given two vectors $\vec{A} = \vec{A}_x i + \vec{A}_y j + \vec{A}_z j$ and $\vec{B} = \vec{B}_x i + \vec{B}_y j + \vec{B}_z j$.

Show that the cross product of the two vectors is given by the determinant of a 3x3 matrix. (4 marks)

h. Prove Lagrange's identity;

$(\mathbf{a}\times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$. (4 marks)

i. State any two axioms of a vector space (2 marks)

QUESTION TWO

a. Use the appropriate differentiation technique to find the first derivative of the following functions

$$f(\theta) = \frac{1+\sin\theta}{\cos\theta}$$

(3 marks)

$$f(x) = (5x^4 - 3x^{-2} + 6x + 11)^{10}$$

(3 marks)

$$f(\theta) = \tan 4\theta e^{k\theta}$$

(3 marks)

b. The parametric equations for the motion of a charged particle released from rest in electric and magnetic fields at right angles to each other take the forms

$x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

Show that the tangent to the curve has slope $\cot\left(\frac{\theta}{2}\right)$. Use this result at a few calculated values of x and y to sketch the form of the particle's trajectory. (11 marks)

QUESTION THREE

a. Apply the appropriate technique to evaluate the following

i. $\int x\sqrt{3x+3}\,dx$ (4 marks)

ii. $\int x^3 e^x\,dx$ (4 marks)

iii. $\int \frac{(x^2-3)dx}{(x^2-1)(x-3)}$ (4 marks)

b. By integrating by parts twice, prove that *In* as defined in the first equalitybelow for positive integers n has the value given in the second equality:

$$I = \int_0^{\frac{\pi}{2}} \sin n\theta \cos\theta d\theta = \frac{n - \sin\left(\frac{n\pi}{2}\right)}{n^2 - 1}$$

(8 marks)

QUESTION FOUR

a. Prove that $\sum_{n=2}^{\infty} \ln\left[\frac{n^r + (-1)^n}{n^r}\right]$ is absolutely convergent for $r = 2$, but only conditionally convergent for $r = 1$. (6 marks)

b. Determine the range of values of x for which the following power series converges

(6 marks)

c. A *Fabry–P´erot* interferometer consists of two parallel heavily silvered glass plates. Light enters normally to the plates, and undergoes repeated reflections between them, with a small transmitted fraction emerging at each reflection.

Find the intensity $|B|^2$ of the emerging wave, where $B = A(1-r)\sum_{n=0}^{\infty} r^n e^{in\phi}$

with r and ϕ being real. (8 marks)

QUESTION FIVE

a. i. Find the angle between the vectors **a** = **i** + 2**j** + 3**k** and **b** = 2**i** + 3**j** + 4**k**. (3 marks)

i. Using the vector method, derive the law of cosines and law of sines (6 marks)

b. In a crystal with a face-centred cubic structure, the basic cell can be taken asa cube of edge *a* with its centre at the origin of coordinates and its edges parallelto the Cartesian coordinate axes; atoms are sited at the eight corners and at thecentre of each face. However, other basic cells are possible. One is the rhomboidwhich has the three vectors **b**, **c** and **d** as edges.

i. Show that the volume of the rhomboid is one-quarter that of the cube. (6 marks)

ii. Show that the angles between pairs of edges of the rhomboid are 60° and that the corresponding angles between pairs of edges of the rhomboid defined bythe reciprocal vectors to **b**, **c**, **d**are each 109.5°. (5 marks)