JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES UNIVERSITY EXAMINATION FOR THEDEGREE OF BACHELOR OF EDUCATION (SCIENCE)

MAIN

REGULAR RESIT

COURSE CODE: SPH 402

COURSE TITLE: STATISTICAL MECHANICS

EXAM VENUE: LAB 1 **STREAM: (BED SCI)**

DATE: 4/5/2016 **EXAM SESSION: 2:00-4:00PM**

TIME: 2 HRS

Instructions:

1. **Answer Question 1 (compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Take where necessary

Gas constant $R = 8.313\, J\, mol^{-1} K^{-1}$

Boltzmann constant $\kappa = 1.38 \times 10^{-38}\, JK^{-1}$

Planck constant $h = 6.63 \times 10^{-34}\, Js$

$\mu_B = 9.27 \times 10^{-24}\, J/T$

SECTION A. (30 MARKS)

ANSWER ALL QUESTION IN THIS SECTION

QUESTION ONE

(a) State any three postulates on which classical thermodynamics is based (3 marks)

(b) Compare and contrast Maxwell-Boltzmann statistics and the Bose-Einstein statistics (2 marks)

(c) Find the number of permutations of the letters in the word **STATISTICAL.** In how many ways are three **T**'s together? In how many ways are (only) two **T**'s together? (4 marks)

(d) State the fundamental difference between the wave functions describing bosoms and fermions. (2 marks)

(e) Briefly explain the following terms as used in describing ensembles

(i) Ensemble (1 mark)
(ii) Micro-canonical ensemble (1 mark)
(iii) Canonical ensemble (1 mark)
(iv) Grand canonical ensemble (1 mark)

(f) If ρ_i is defined as the probability of n_i molecules in the ith energy state, prove that the entropy will be

$$S = -k\sum_i \rho_i \ln \rho_i$$

(4 marks)

(g) Show that, if the number of particles n_i in the most populated level is less than the number of states g_i in the level i, then the three equations for the different distribution laws become equivalent to the Boltzmann distribution law. (4 marks)

(h) In how many ways can 9 distinguishable particles be distributed among four energy levels such that $n_0 = 2, n_1 = 3,\ n_2 = 3$ and $n_3 = 1$ if their respective degeneracy's are $g_0 = 2, g_1 = 3,\ g_2 = 2$ and $g_3 = 1$. (3 marks)

(i) The mass of hydrogen molecule in a cubic container of dimensions $1cm$ is $3.2\times10^{-27}\,kg$. Calculate the average value $\left(n_x^2 + n_y^2 + n_z^2\right)$ for hydrogen at $27^0 C$.

(4 marks)

SECTION B (ANSWER ANY TWO QUESTIOS. EACH QUESTION CARRIES 20 MARKS.)

QUESTION TWO

2 (a) Stirling's two-term and three-term approximation are given by

$$\ln N! = N \ln N - N \quad (1)$$

and

$$\ln N! = N \ln N - N + \ln\sqrt{2\pi N} \quad (2)$$

respectively.

(i) Calculate $\ln N!$ for $N = 10,\ N = 500$ and $N = 1000$ using the expressions in equations (1) and (2) separately and comment on the results. (6 marks)

(ii) Using the results in 2 (a) (i) calculate the total number of distributions for a system consisting of $N_1 = N_2 = 500$ particles. (3 marks)

2 (b) Figure 1 shows four possible arrangements of an assembly of six particles obeying Fermi-Dirac statistics. The energy levels are equally spaced and have a degeneracy of $g_j = 3$ each. The total energy of the system is $U = 6\varepsilon$.

	$k=1$	$k=2$	$k=3$	$k=4$
4ε	●			
3ε		●	●	
2ε		●		●●
ε	●●	●	●●●	●●
0	●●●	●●●	●●	●●

Fig. 1 The five possible macrostates of an assembly of six particles obeying Fermi-Dirac statistics.

Calculate

(i) the thermodynamic probability of each macrostate (4 marks)

(ii) the total number of macrostate of the assembly (1 mark)

(iii) the average occupation number of each level (5 marks)

(iv) the sum of the average occupation numbers (1 mark)

QUESTION THREE

3 (a) (i) A system is always assumed to behave as though it is in its most probable configuration with the number of distributions given by

$$W_{\max} = N!\Pi_j \frac{g_j^{n_j}}{n_j!}$$

where N is the number of particles, n_j is the most probable configuration population number and g_j the degeneracy. Starting from this equation derives the Boltzmann distribution function. (11 marks)

(ii) Calculate the ratio of molecules at $27^{\circ}C$ in energy states separated by energy equal to $8.314\ kJ \cdot mol^{-1}$. (3 marks)

3 (b) (i) Write down the number of ways of obtaining a distribution in which we have n particles of type A and $(N-n)$ particles of type B on one side of a system and $(N-n)$ particles of type A and n particles of type B on the other. (2 marks)

(ii) Show that the entropy of the system in 3 (b) (i) is given by

$$S = 2\kappa\left(N\ln N - n\ln n - (N-n)\ln(N-n)\right)$$

(4 marks)

QUESTION FOUR

4. (a) The probability ρ_i that at any instance that a particle can be found associated with energy ε_i in a grand canonical ensemble is

$$\rho_i = \frac{\exp\left(-\varepsilon_i/\kappa T + \mu N/\kappa T\right)}{\Xi(T,V,\mu)}$$

where $\Xi(T,V,\mu)$ is the grand canonical partition function. Obtain the expression of the following statistical thermodynamic functions in terms of the of the grand canonical partition function $\Xi(T,V,\mu)$

(i) Grand canonical potential $F(T,V,\mu)$ (5 marks)

(ii) Pressure $p(T,V,\mu)$ (1.5 marks)

(iii) Entropy $S(T,V,\mu)$ (2 marks)

(iv) Number of particles $N(T,V,\mu)$ (1.5 mark)

(b) The canonical partition function $Z(N,V,T)$ for N distinguishable particles of the ideal gas is

$$Z(T,V,N) = \frac{V^N}{N!\lambda^{3N}}$$

where $\lambda = \left(\frac{h^2}{2\pi m\kappa T}\right)^{1/2}$ is the thermal wavelength. Calculate the following statistical thermodynamic functions for the ideal gas

(i) Helmhotz free energy $F(T,V,N)$ (4 marks)

(ii) Pressure $p(T,V,N)$ (2 marks)

(ii) Entropy $S(T,V,N)$ (4 marks)

QUESTION FIVE

(a) (i) Show that the net magnetic moment M of an assembly of N distinguishable magnetic dipoles is given by

$$M = \left(\bar{N}_{\uparrow} - \bar{N}_{\downarrow}\right)\mu_B = N\mu_B \tanh\frac{\mu_B H}{\kappa T}$$

where $\bar{N}_{\uparrow}$ and $\bar{N}_{\downarrow}$ represents the average occupation number of ions whose moments are aligned parallel and antiparallel to the field H . (8 marks)

(ii) Show that the net magnetic moment follows Curie's law in the limit of high temperatures and weak fields. (2 marks)

(iii) Show that dipoles are aligned in the limit of low temperature and strong fields. (2 marks)

(b) The canonical partition function of the assembly of three dimensional oscillators is given by

$$Z(T,V,N) = \frac{\exp\left(-3\Theta/2T\right)}{\left(1-\exp\left(-\Theta/T\right)\right)^{3}}$$

where $\Theta = hv/\kappa$ is the characteristic temperature. Obtain the expression for mean energy and heat capacity of the assembly. (8 marks)