Jaramogi Oginga Odinga University of Science and Technology

UNIVERSITY *DRAFT* EXAMS 2015/2016

SCHOOL OF INFORMATICS

*SEMESTER ONE, FIRST   YEAR **BSC** EXAMINATIONS*

SMA3114: ANALYTICAL COMPUTATIONAL METHODS

May, 2016   Time 2hours

Instructions

Answer Question1 and TWO other questions.

Show all the necessary working

Question1 [30 marks] COMPULSORY

(a)E valuate (i) $\lim\limits_{x\to 2}\left\{\dfrac{x-14}{x+14}\right\}$ (ii) $\lim\limits_{x\to -1}\left\{\dfrac{x^2-1}{x-1}\right\}$ (iii) $\lim\limits_{h\to 0}\left\{\dfrac{(x+2h)^2-(x-2h)^2}{4h}\right\}$ [4 marks]

(b) Using primitive definition of derivative, determine

$\dfrac{dy}{dx}$ given, $y(x)=x^2-2x-4$ [5 marks]

(c) (i) Solve $\sin\theta=\tfrac{1}{2}$ for $0°\le\theta\le 720°$ [5 marks]

(ii)Find x if $5\sin x-\cos x=1$: $0°\le x\le 360°$ [5 marks]

(d) Evaluate the indefinite integral (i) $\int 4x^2dx$ (ii) $\int\left(\dfrac{1}{x^2}+\cos x\right)dx$ (iii) $\int\dfrac{2+4x}{x+x^2}dx$ [6 marks]

.(e) If $Z=2+2i$ express the complex numbers $\overline{Z}$, $\overline{Z}Z$ in the form $a+ib$. [5 marks]

(f) The distance S in meters, of a particle traveled from a point Q after time t seconds, is given by

$S=\dfrac{t^3}{3}-144t+210$. When is the particle instantaneously at rest? Determine the speed and

acceleration of the particle after12 seconds. [7 marks]

Question2 [20 marks]

(a) (i) Prove that $\dfrac{d}{dx}\big(u(x)v(x)\big)=v(x)\dfrac{du(x)}{dx}+u(x)\dfrac{dv(x)}{dx}$ [4 marks]

(ii) If $y=(13+x^2)e^x$ find $\dfrac{dy}{dx}$, at $x=0$ [5 marks]

(b) (i) Given $y=\log_e(3+4x^2)$ find $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$, at $x=1$ [6 marks]

(c) Determine $\dfrac{dy}{dx}$, if $y=\left[\dfrac{3+x^2}{e^{\alpha x}}\right]^{-1}$ [5 marks]

Question3 [20 marks]

Define a curve $y(x)$ by $y(x)=x^3-48x+25$.

(i)Find $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ (ii) Determine and distinguish the turning points of the curve.

(iii) Sketch the curve $y=x^3-48x$; $-150\le x\le 150$ [15 marks]

(iv) State both the minimum and maximum values of the $y=x^3-48x$; $-150\le x\le 150$ [5 marks]

Question4 [20 marks]

.(a.) If $Z_1 = \dfrac{1}{2} + \dfrac{1}{2}i$, $Z_2 = 2 - 2i$ express the complex numbers , Z_1^{10}, Z_2^{10} in the form

$Z = r(\cos\theta + i\sin\theta)$.   [6 marks]

 (b) On same Argand plane, plot the complex numbers Z_1 , Z_1^{10}, Z_2 , Z_2^{10} .

     Determine the effect of raising a complex number to a positive power.   [6 marks]

(c) Find x if $13\sin x - \cos x = 1$: $0° \le x \le 720°$ [8 marks]

Question5 [20 marks]

(a) Given the matrices $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $C = \begin{bmatrix} 16 \\ 10 \\ 0 \end{bmatrix}$
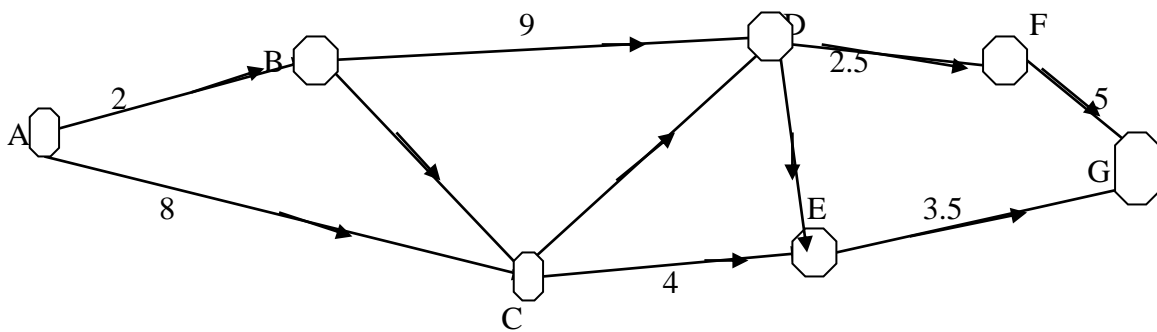
 (i). Compute   det A

(ii). Show that det $A \ne 0$

(iii) If $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{5}{2} & -\frac{7}{2} \\ \frac{1}{2} & \frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{5}{2} & -\frac{7}{2} \\ \frac{1}{2} & \frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} = P$, find matrix P and give the inverse

of A .

 (iv) Using the results of part (iii) above solve the equation $AX = C$.   [15 marks]

 (c) Find the shortest path from source to sink on the system below.

                                                            [4 marks]

(d) Determine the constant term in the binomial expansion $\left(x - \dfrac{15}{x} \right)^{10}$ [4 marks]