JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2^{TH} YEAR 1^{ST} SEMESTER 2015/2016 ACADEMIC YEAR

MAIN CAMPUS - RESIT

COURSE CODE: SMA 100

COURSE TITLE BASIC MATHEMATICS

EXAM VENUE: LAB 1 **STREAM: (BSc. Actuarial/BED)**

DATE: 04/5/16 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question1[30 marks] COMPULSORY

(a) (i) Given $g(m)=8^m-1$, express $g(1)$, $g(2)$, $g(3)$ as multiples of 7 .

(ii) Prove that $g(m)=8^m-1 : m=1,2,3,......$ is divisible by 7 [8 marks]

(b) (i) Solve sin θ = $\frac{1}{2}$ for $0° \le \theta \le 360°$

(ii) Solve cos 2θ = $\frac{1}{2}$ for $0° \le \theta \le 360°$ [5 marks]

(c) Evaluate 5 *in* mod(4) , 5^6 *in* mod(7) and 12^{15} *in* mod(15) [7 marks]

(d) Determine the constant term in the binomial expansion $\left(2x-\frac{5}{x}\right)^{20}$ [5 marks]

Simplify the value of the constant.

(e) (i) Determine the value of $\log_{21} 2150$

(ii) Simplify $\log_{21} 9261+\log_{194481} 21$ [5 marks]

Question2 [20 marks]

.(a.) If $Z_1=\frac{1}{2}+\frac{1}{2}i$, $Z_2=2+2i$ express the complex numbers Z_1 , Z_1^{10}, Z_2 , Z_2^{10} in the form $Z=r(\cos\theta+i\sin\theta)$. [6 marks]

(b) On same Argand plane, plot the complex numbers Z_1 , Z_1^{10}, Z_2 , Z_2^{10}. Determine the effect of raising a complex number to a positive power. [4 marks]

(c) Define a relation $\sqcup$ on Z ,the set of integers, by m □ n if $m-n$ is divisible by 5 .Show that

(i) $\sqcup$ is reflexive (ii) $\sqcup$ is symmetric (iii) $\sqcup$ is transitive

(iv) $\sqcup$ defines an equivalence relation on Z [10 marks]

Question3 [20 marks]

(i) Show that $\sin(x+y)-\sin(x-y)=2\sin y\cos x$ [3 marks]

(a)

(ii) Prove that $\frac{\sin(2x+x)}{1+2\cos 2x}=\sin x$ [4 marks]

(b) Given the multiplication in $Z/7$ as shown below.

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2					
3	3		2			
4	4					
5	5				4	
6	6					1

(i) Complete the multiplication table

(ii) Show that $(5\times4)\times6=5\times(4\times6)$

(iii) State the neutral element

(iv) Determine the inverse of each element. [13 marks]

Question4 [20 marks]

(a) Given the matrices $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, C = \begin{bmatrix} 16 \\ 0 \\ 8 \end{bmatrix}$

(i). Compute det A

(ii). Show that det $A \neq 0$

(iii). Express the augmented matrix for the system $AX = B$ as $\left| A \quad \middle| B \right|$

(iv). Row reduce the augmented matrix till you obtain $\left| \quad I_{3x3} \quad \middle| \begin{matrix} \alpha_{11} a + \alpha_{12} b + \alpha_{13} c \\ \alpha_{21} a + \alpha_{22} b + \alpha_{23} c \\ \alpha_{31} a + \alpha_{32} b + \alpha_{33} c \end{matrix} \right|$

(v) Show that $\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} = I_{3x3}$ *the identity matrix*, and give the inverse of A.

(vi) Using part (iv) above solve the equation $AX = C$. [15 marks]

(b) Find x if $\sqrt{3}\sin x - \cos x = 1 \; : \; 0° \leq x \leq 360°$ [5 marks]

Question5[20 marks]

(a) Expand $(1+x)^{1/2}$ as far as x^5 term. Use a suitable value of x in the expansion of $(1+x)^{1/2}$ to approximate the value of $\sqrt{105}$ correct to five decimal places. Compute the relative percentage error in the approximation of $\sqrt{105}$. [12 marks]

(b) Prove by induction that $1^3 + 2^3 + 3^3 + ... + n^3 = (1+2+3+....+n)^2 : \; n = 1, 2, 3, ...$ [8 marks]