JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2TH YEAR 1ST SEMESTER 2015/2016 ACADEMIC YEAR

MAIN CAMPUS - RESIT

COURSE CODE: SMA 201

COURSE TITLE: LINEAR ALGEBRA II

EXAM VENUE: LAB 1 **STREAM: (BSc. Actuarial/BED)**

DATE: /5/16 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question1 [30marks] Compulsory

(a) Define a linear mapping T from an m-dimensional vector space X into n-dimensional vector space Y over the real field F. Let $B = \{u_1, u_2, u_3, ..., u_m\}$, $\Psi = \{v_1, v_2, v_3, ..., v_n\}$ be the bases for the respective vector spaces X and Y. Construct matrix M of T with respect to the ordered bases B, Ψ. [10 marks]

(b) Let $P = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 2 & 1 \\ -2 & 1 & 2 \end{pmatrix}$ be a real square matrix.

Prove that P is orthogonal hence find P^{-1}, and $\hat{P}$ the orthonormalized form of P. **[10marks]**

(c) Let U be a vector space over field F of complex numbers.

Suppose $a, b, c, d \in U;\ k, l \in F$.

(i) Define a rule $*$, on U together with F for which $*$ is known as an inner product on U.

(ii) Show that the rule $\oplus$, defined on the R^2 vector space by : $\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = xx_1 + yy_1$ is an inner product. . **[10marks]**

Question2 [20marks]

Given the vectors $b_1 = [3, 0, 4]$, $b_2 = [-1, 0, 7]$, $b_3 = [2, 9, 11]$ **of vector space R^3 with the standard inner product.**

(i) Show that $b_1 = [3, 0, 4]$, $b_2 = [-1, 0, 7]$, $b_3 = [2, 9, 11]$ are linearly independent.

(ii)Apply the Gram-Schmidt process to the vectors $b_1 = [3,0,4]$, $b_2 = [-1,0,7]$, $b_3 = [2,9,11]$ to obtain the corresponding orthogonal set of vectors a_1, a_2, a_3 given .

Question3 [20marks]

Let W be the space of all 3×3 matrices A over R which are skew- symmetric $i.e., A^t = -A$. We equip W with the inner product $[A*B] = \frac{1}{2} tr[AB^t]$. Let V be the vector space R^3 with the standard inner product. If T be the mapping from V into W defined by

$$T(x, y, z) = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \quad i.e.\ T: V \to W$$

(a)Show that $T(0_V) = 0_W$

(b)Show that T is linear

Question4 [20marks]

Define a linear mapping T from an m-dimensional vector space X into n-dimensional vector space Y over the real field F . Let $B = \{u_1, u_2, u_3, ..., u_m\}$, $\Psi = \{v_1, v_2, v_3, ..., v_n\}$ be the bases for the respective vector spaces X and Y .

(i)Construct matrix M_T of T with respect to the ordered bases B, Ψ . [12marks]

(ii) Determine the kernel of T [8marks]

Question5 [20marks]

(a)The matrix $A = \begin{pmatrix} 8 & -2 & -3 & 1 \\ 7 & -1 & -3 & 1 \\ 6 & -2 & -1 & 1 \\ 5 & -2 & -3 & 4 \end{pmatrix}$ of linear transformation T has eigenvectors $v_1 = [1, 1, 1, 1]^t$, $v_2 = [1, 1, 1, 0]^t$, $v_3 = [2, 5, 2, 2]^t$. Determine all the eigenvalues [13 marks]

Determine all the remaining eigenvectors of T . [13 marks]

(b) Diagonalize matrix A [7 marks]