

**JARAMOGI OGINGA ODINGA UNIVERSITY OF
SCIENCE AND TECHNOLOGY**

UNIVERSITY *DRAFT* RESIT/RETAKE EXAMS 2015/2016

SEMESTER TWO SECOND YEAR BEd

SMA202: VECTOR ANALYSIS

August, 2012

Time: 2hrs

INSTRUCTIONS

Answer **Question1** and **two** other questions
 Show all the necessary working

Question.1 [30 marks] Compulsory

(a) Given the vectors $\underline{u} = 3i - 12j + k$, $\underline{v} = 12i - 4j - 3k$, $\underline{w} = -i + 2j + 2k$, find the magnitudes of

(i) $7\underline{u}$ (ii) $-2\underline{u} + 3\underline{v} + 5\underline{w}$ [6 marks]

(b) Determine the minimum turning point of the following scalar function

$$\Phi(x, y) = x^3 + y^2 - 3(x + y) + 101 \quad [8 \text{ marks}]$$

(c) If $\phi(x, y, z) = x^2 y^2 z$, $\underline{a} = xz i - xy^2 j + yz^2 k$, find $\frac{\partial^4}{\partial y \partial x^2 \partial z} [\phi \underline{a}]$ at the point $(0, 1, 1)$. [8marks]

(d) Determine which of the vectors (i) $\underline{v} = 50y\underline{i}$ (ii) $\underline{v} = \frac{50}{(x^2 + y^2)}(-y\underline{i} + x\underline{j})$ (iii) $\underline{v} = 50(-y\underline{i} + x\underline{j})$

are; incompressible, irrotational. [8marks]

Question.2 [20 marks]

(a) For the vectors $\underline{\alpha} = 15t^2 i + t j - t^3 k$, $\underline{\beta} = \sin t i - \cos t j + 4k$, find $\frac{d(\underline{\beta} \times \underline{\alpha})}{dt}$. [9 marks]

- (b) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2 \cos 3t$, $z = 2 \sin 3t$ where t is the time. Determine the components of the velocity and acceleration at $t = 0$ in the direction $\underline{w} = i - 3j + 2k$. [11 marks]

Question.3 [20 marks]

Determine and discuss the nature of the extrema of the following scalar function

$$\Phi(x, y, z) = 8x^2 + 24y^2 + 16z^2 + 24x + 16z + 144 \quad [20 \text{ marks}]$$

Question.4 [20 marks]

- (a) If a vector field $\underline{F}(x, y, z)$ is conservative then show that the work integral

$$\int_C \underline{F} \cdot d\underline{r}$$

between points A and B is independent of the path C chosen between the two points. [6 marks]

- (b) Compute $\text{curl } \underline{F} : \underline{F}(x, y, z) = (z^3 y^2 \cos x - 4x^3 z) \underline{i} + 2yz^3 \sin x \underline{j} + (3z^2 y^2 \sin x - x^4) \underline{k}$ [6 marks]

- (c) Find the work done in moving a particle through straight lines, from $O(0,0,0)$ to $Q(-1,-1,10)$ then to $R(1,1,1)$ in the force field $\underline{F}(x, y, z)$ above. [8 marks]

Question.5 [20 marks]

Evaluate the double integral $\iint_S (\nabla \times \underline{F}) \cdot \underline{n} dS$ for $\underline{F} = (y - z + 2) \underline{i} + (zy + 4) \underline{j} - xz \underline{k}$ where

S is the surface of the cube $x = 0, y = 0, z = 0, x = 140, y = 140, z = 140$

with XY plane face missing. [20marks]