JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2TH YEAR 1ST SEMESTER 2015/2016 ACADEMIC YEAR

MAIN CAMPUS - RESIT

COURSE CODE: SMA 210

COURSE TITLE PROBABILITY AND DISTRIBUTION THEORY I

EXAM VENUE: LAB 1 **STREAM: (BSc. Actuarial/BED)**

DATE: 04/5/16 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1:[30 marks] COMPULSORY

Q1(a) A random variable X has a distribution with its probability density function given as:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} .$$

Identify the distribution of X and the parameters μ and σ. (4 marks)

Q1 (b) Let $X \sim N(10, 25)$, compute

(i) $P(X \leq 20)$ (5 marks)

(ii) $P(X > 5)$ (6 marks)

(iii) $P(12 \leq X \leq 15)$ (4 marks)

Q1 (c) The joint probability density function of X and Y ,is

$f_{XY}(x, y) = k(1 - x - y)$, $x + y \leq 1, 0 < x < 1, 0 < y < 1$.

Obtain

the marginal probability density function of X (4 marks)

Q1 (d) Consider a Poisson distributed random variable X with probability density function

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \ \lambda > 0 \quad x = 0,1,2,.......$$

having the moment generating function, $M_X(t) = e^{-\lambda} e^{\lambda e^t}$

Verify that for such variable X , the $E(X) = E(X - \mu)^2$ (7 marks).

Question 2: [20 marks]

(a) Let X, be a Normally distributed random variable with parameters μ and σ .

(i) Prove that the moment generating function of X is: $M_X(t) = e^{\mu t} + \frac{\sigma^2 t^2}{2}$, (8 marks)

(ii) Calculate $E(X)$, $Var(X)$ (7 marks)

(b) Suppose $X \sim N(\alpha, \sigma_x^{\ 2})$ and $Y \sim N(a, \sigma_y^{\ 2})$ and that X and Y are independent

Determine the distribution of $U = aX + bY$.

(5marks)

Question 3: [20 marks]

(a) Consider a Poisson distributed random variable X with probability density function

$$f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \ \lambda > 0 \quad x = 0,1,2,...$$

having the moment generating function, $M_X(t) = e^{-\lambda} e^{\lambda e^t}$

Verify that for such a variable, the mean and variance are equal. (8 Marks).

(b) Suppose that flaws in plywood occur at random with an average of one flaw per 50cm2.What is the probability that a 3 cm by 8cm sheet will have:

(i) no flaws (3 Marks)

(ii) at most one flaw? (3 Mark)

(iii) at least one flaw? (3 Mark)

(iv) between 1 and 4 flaws? (4 Marks)

Question4: [20 marks]

Let $U, V, \text{ and } W$ be identical independently distributed(i.i.d) $G(1, \beta)$ random variables. Compute

(i) the joint p.d.f for $R = U$, $S = U + V$ and $T = U + V + W$ (8 marks)

(ii) the Jacobian of the transformation (4 marks)

(iii) the relationship between the variables, S, R, T (2 marks)

(iv) the p.d.f for, S (6 marks)

Question 5:[20 marks]

Let (X,Y) have a bivariate normal distribution with parameters $\mu_x, \mu_y, \sigma_x^2, \sigma_y^2$ and ρ ,

where the joint moment generating function is $M_{X,Y}(t_1, t_2) = e^{t_1\mu_x + t_2\mu_y + \frac{\left[(t_1\sigma_x)^2 + 2t_1t_2\rho\sigma_x\sigma_y + (t_2\sigma_y)^2\right]}{2}}$

for all values of $t_{1,}\, t_{2.}$

(a) Determine the moment generating functions $M_X(t_1)$, $M_Y(t_2)$ for X and Y respectively. (10marks)

(b) A random variable X is said to have a gamma distribution with parameters α , β ,denoted by $X \sim G(\alpha, \beta)$,if it has a probability density function of the form

$$f_X(x) = \begin{cases} \dfrac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^{\alpha}}, 0 < x, 0 < \alpha, 0 < \beta \end{cases}$$,with moment generating function $M_X(t) = \left(\frac{1}{1 - \beta t}\right)^{\alpha}$

Suppose $X \sim G(\alpha, \beta)$ and $Y \sim G(\eta, \beta)$, and that X and Y are independent.

Determine the distribution of $S = 8Y + 8X$ and obtain its probability density function. (10 marks)