JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2ND YEAR SPECIAL RESITS – 2016

MAIN REGULAR

COURSE CODE: SMA 211

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY II

EXAM VENUE: LAB 1 **STREAM: (BSc. Actuarial)**

DATE: 06/05/2016 **EXAM SESSION: 2.00 – 4.00 PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1:[30 marks] COMPULSORY

Q1(a) A random variable X has a distribution with its probability density function given as:

$$f_X(x) = \frac{1}{\sqrt{50\pi}} e^{\frac{-(x-20)^2}{50}} \ldots$$

Identify the distribution of X and state its mean and the variance. (7 marks)

Q1 (b) Consider a take-home salary of a Nakummat supermarket account's clerk with sample data: n = 100, $\bar{X}$ = Kshs 14,000 , s = Kshs 1,000

Construct a 95%, Confidence Interval Estimator of the expected take-home salary. Give the Interpretation of your results. (8 marks)

Q1(c) Let U_1 *and* U_2 be independent χ^2 square random variables with degrees of freedom k_1 *and* k_2 respectively.

Then $Y = \frac{U_1}{U_1 + U_2}$ $\sim Beta\left(\frac{k_1}{2}, \frac{k_2}{2}\right), \frac{k_1}{2} = \alpha_1, \frac{k_2}{2} = \alpha_2$)

Write the pdf of Y. $f_Y(y) = \frac{y^{\alpha_1 - 1}(1-y)^{\alpha_2 - 1}}{\beta(\alpha_1, \alpha_2)}, \quad 0 < y < 1$ (7 marks)

Q1 (d) A random variable X has moment generating function $M_X(t) = \left(\frac{1}{1-\beta t}\right)^{\alpha}$, with parameters α , β . Determine mean and variance of X . (8 marks)

Question 2:[20 marks]

Let (X, Y) have a bivariate normal distribution with parameters $\mu_x, \mu_y, \sigma_x^2, \sigma_y^2$ and ρ ,

where the joint moment generating function is $M_{X,Y}(t_1, t_2) = e^{t_1\mu_x + t_2\mu_y + \frac{\left[(t_1\sigma_x)^2 + 2t_1t_2\rho\sigma_x\sigma_y + (t_2\sigma_y)^2\right]}{2}}$

for all values of t_1, t_2.

(a) Determine the moment generating functions $M_X(t_1), M_Y(t_2)$ for X and Y respectively. (8marks)

(b) Given that X ,Y are independent

(i) State the significance and value of the parameter ρ

(ii) Obtain the joint moment generating function $M_{X,Y}(t_1, t_2)$.

(iii) Obtain the joint pdf of X, Y. (12 marks)

Question 3: [20 marks]

(a) A company claims that a box of its raisin bran cereal contains at least 100 raisins. An inspector working for the Federal Trade Commission takes a random sample of 324 boxes of cereals and finds that: $\bar{X}$ = 97 raisins and s = 9 raisins. Should the

company's claim be rejected? Test at $\alpha = 0.05$.

H_0: $\mu \geq 100$

H_1: $\mu < 100$ (10 marks)

(b) Compare death rates of liver transplants at 2 hospitals. Test at α=.05

Hospital A; 77patients out of 100, died within 6 months

Hospital B; 120 patients out of 200 died within 6 months (10 marks)

Question 4: [20 marks]

Determine the relationship between hours studied and grade on a quiz on seven pairs of data. Maximum grade on quiz is 15.

X is hours studied; Y is grade on quiz.

X	1	2	3	4	5	6	7
Y	5	8	9	10	11	12	14

(a)Calculate

(i) the correlation coefficient, r .

(ii) the coefficient of determination.

(iii) the regression coefficient b_1 (the slope).

(iv) the regression coefficient b_0 (the Y-intercept, or constant). (10 marks)

(b)Explain the meaning of the regression coefficients. (5 marks)

(c) If someone studies 5.3 hours, what would we predict his/her quiz score to be? (5 marks)

Question 5: [20 marks]

In a sample of 16 management consultants, the following statistics relating to hourly wages have been computed:

$\bar{X}$ = \$200 , s = \$96

(a) Test at α = .05 that μ = \$260. (11 marks)

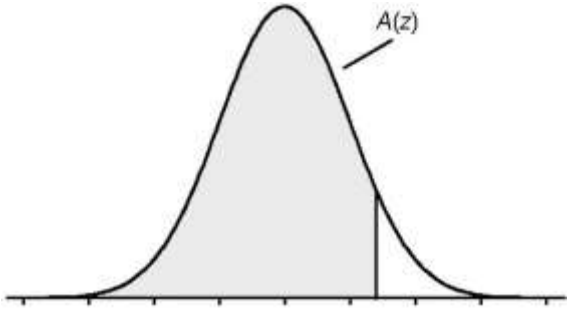
(b) Construct a 95%, Confidence Interval Estimator of μ. (9 marks)

The Short student t Table: ***Critical Values of t***

Degrees of Freedom	*Upper Tail Areas*					
	.25	*.10*	*.05*	*.025*	*.01*	*.005*
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3 534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4 759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
10	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
11	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
12	0;6955	1.3562	1. 7823	2.1788	2.6810	3.0545
13	0.6938	1.3502	1.7709	2.1604	2.6503	3.0123

14	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
15	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467
16	0.6901	1.3368	1.7459	2.1199	2.5835	2.9208
17	0.6892	1.3334	1.7396	2.1098	2.5669	2.8982
18	0.6884	1.3304	1.7341	2.1009	2.5524	2.8784
19	0.6876	1.3277	1.7291	2.0930	2.5395	2.8609
20	0.6870	1.3253	1.7247	2.0860	2.5280	2.8453
21	0.6864	1.3232	1.7207	2.0796	2.5177	2.8314
22	0.6858	1.3212	1.7171	2.0739	2.5083	2.8188
23	0.6853	1.3195	1.7139	2.0687	2.4999	2.8073
24	0.6848	1.3178	1.7109	2.0639	2.4922	2.7969
25	0.6844	1.3163	1.7081	2.0595	2.4851	2.7874
26	0.6840	1.31 50	1.7056	2.0555	2.4786	2.7787
27	0.6837	1.3137	1.7033	2.0518	2.4727	2.7707
28	0.6834	1.3125	1.7011	2.0484	2.4671	2.7633
29	0.6830	1.3114	1.6991	2.0452	2.4620	2.7564
30	0.6828	1.3104	1.6973	2.0423	2.4573	2.7500
31	0.6825	1.3095	1.6955	2.0395	2.4528	2.7440
32	0.6822	1.3086	1.6939	2.0369	2.4487	2.1385
33	0.6820	1.3077	1.6924	2.0345	2.4448	2.7333
34	0.6818	1.3070	1.6909	2.0322	2.4411	2.7284
35	0.6816.	1.3062	1.6896	2.0301	2.4377	2.7238
36	0.6814	1.3055	1.6883	2.0281	2.4345	2.7195
37	0.6812	1.3049	1.6871	2.0262	2.4314	2.7154
38	0.6810	1.3042	1.6860	2.0244	2.4286	2.7116
39	0.6808	1.3036	1.6849	2.0227	2.4258	2.7079
40	0.6807	1.3031	1.6839	2.0211	2.4233	2.7045
41	0.6805	1.3025	1.6829	2.0195	2.4208	2.7012
42	0.6804	1.3020	1.6820	2.0181	2.4185	2.6981
4J	0.6802	1.3016	1.6811	2.0167	2.4163	2.6951
...	...	...	...	...	...	...
∞			**1.645**	**1.96**	**2.33**	**2.575**

Cumulative Standardized Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830

1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							