JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY ***DRAFT*** **RESIT/RETAKE EXAMS 2015/2016**

SEMESTER II THIRD YEAR BSc+ BEd

SMA302: LINEAR ALGEBRA III

DATE : May,2016 TIME: 2hrs

INSTRUCTIONS

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Answer **Question1** and **two other** questions
Show all the necessary working

Question1 **[30 marks] COMPULSORY**

(a) Let U be a linear operator on an inner product space V .Then prove that U is unitary if and only if the adjoint U^* of U exists and $U^*U = UU^* = I$ [**10 marks]**

(b) A form on a real vector space V is a real valued function f on $V \times V$ with values in R

State the axioms satisfied by f so that it is bilinear form. [**5 marks]**

(c) Let f be the form on $V \times V$ such that V is a real vector space. Define $A = \left(a_{ij}\right)$, the matrix of f w.r.t an ordered basis $\beta = \{\beta_1, \beta_2,,, \beta_n\}$.

Suppose f is a form on R^2 defined by

$f\left(\left(x_1, x_2\right), \left(y_1, y_2\right)\right) = x_1y_1 + 4x_2y_2 + 2x_1y_2 + 2x_2y_1$.

Find the matrix of f in each of the bases

(i) $\{[1,0],[0,1]\}$ (ii) $\{[1,-1],[1,1]\}$ [**15 marks]**

Question2[20 marks]

Find the principal values and the orthonormal principal axes the of stress if the stress tensor is

$T_{i,j} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$. **[20 marks]**

Question3 [20 marks]

Consider R^4 with the inner product

$\langle x, y\rangle = \frac{1}{2}x_1y_1 + \frac{1}{2}x_2y_2 + x_3y_3 + x_4y_4$ **;** $x = [x_1, x_2, x_3, x_4]$, $y = [y_1, y_2, y_3, y_3]$**,** $x, y \in R^4$

(a)Show that the vectors $\{v_1 = [1,1,-1,-1], v_2 = [1,1,1,1], v_3 = [-1,-1,-1,1], v_4 = [1,0,0,1]\}$ are linearly independent. **.** **[5 marks]**

(b)Apply the Gram-Schmidt process to the vectors

$v_1 = [1,1,-1,-1], v_2 = [1,1,1,1], v_3 = [-1,-1,-1,1], v_4 = [1,0,0,1]$

to obtain orthogonal basis $\{w_1, w_2, w_3, w_4\}$**.** **[10 marks]**

(c) Obtain an orthonormal basis of R^4**.** **[5 marks]**

Question4 [20 marks]

Let W be the space of all three by three matrices A over R which are skew symmetric i.e. $A^t = -A$ .We equip W with the inner product $\langle A, B\rangle = \frac{1}{2} trace\left(AB^t\right)$. Let V be the space R^3 with the standard inner product $\langle [x_1, x_2, x_3] \otimes [y_1, y_2, y_3]\rangle = x_1y_1 + x_2y_2 + x_3y_3$. Let T be the linear transformation from V into W defined by

$$T[x_1, x_2, x_3] = \begin{bmatrix} 0 - x_3 & x_2 \\ x_3 & 0 - x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

(a) Show that T is a vector space isomorphism of V onto W **..[8 marks]**

(b)Given $\{e_1, e_2, e_3\}$ the standard basis for R^3,show that the set $\{Te_1, Te_2, Te_3\}$ is a basis for W consisting of the matrices

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$ **[7 marks]**

(c)Verify that $\{Te_1, Te_2, Te_3\}$ is orthonormal. **.[5 marks]**

Question5[20 marks]

For a given stress tensor $T_{i,j}$; $T_{i,j} = T = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & -1 \\ 4 & -1 & 1 \end{bmatrix}$

(a) find the three basic tensor invariants $I^{(1)}{}_T$, $I^{(2)}{}_T$, $I^{(3)}{}_T$

[12 marks

(b)show that $I^{(1)}{}_T$, $I^{(2)}{}_T$, $I^{(3)}{}_T$ are truly invariant when the tensor is subjected to a rotation with direction cosine matrix of $l_{i,j} = L = \begin{bmatrix} \frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}$. **[8 marks]**