JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY ***DRAFT*** **RESITRETAKE EXAMS 2015/2016**

SCHOOL OF MATHEMATICS ,APPLIED STAT. AND ACTUARIAL SCIENCES

SEMESTER 2 THIRD YEAR Bed EXAMS

SMA303: COMPLEX ANALYSIS 1 **Time 2hours**

Instructions

Answer **Question1** and **two** other questions.

Show all the necessary working

Question1 [30 marks] Compulsory

1(a) (i) Show that $\frac{e^{i\theta}+e^{-i\theta}}{2}=\cos\theta,\quad \frac{e^{i\theta}-e^{-i\theta}}{2i}=\sin\theta$ [4 marks]

(ii) Express $(-1)^{1/4}$ in rational Cartesian form. [6 marks]

[10 marks]

(b) Prove that $\left[\cos\theta+i\sin\theta\right]^{m}=\cos m\theta+i\sin m\theta: m=0,1,2,...$ [3marks]

(c) Determine the residues of $f(z)=\frac{1}{z^2+4z+3}$ [4marks]

(b) (i) Find the $\lim_{z\to 10+i}\left\{\frac{z^3+2z-4+i}{z+i+2}\right\}$ [2 marks]

(ii) Suppose $f(z)=z^2$ and $\Delta z=z-z_0$ **,** determine the $\lim_{\Delta z\to 0}\left\{\frac{f(z)-f(z_0)}{\Delta z}\right\}$

and hence find $f'(z_0)$. [5 marks]

Question 2 [20 marks]

(a) If $f(z)=z\bar{z}$ find $\lim_{z\to z_0}\left\{\frac{f(z)-f(z_0)}{z-z_0}\right\}$. Discuss the existence $f'(z_0)$, the derivative of $f(z)$ on the complex plane. [9 marks]

(b) Find all the points at which the function $f(z)=x^3-i(1-y)^3$ is differentiable. [11 marks]

Question 3 [20 marks]

(a) Compute the integral of $f(z)=(x^2+y)+i(xy),$ *from* $a=0$ *to* $b=1+i$ along the path of parabola $C:\gamma(t)=t+it^2,\ 0\le t\le 0$ [10marks]

(b)Evaluate the integral $\oint_{|z|=3} \frac{\sin z}{z(z-1)}dz$ where the contour of integration is the circle centre at 0 and with radius 3 followed in the positive (anticlockwise) direction. [10marks]

Question 4 [20 marks]

(a) Evaluate the integral $\int_C z^2 dz : C \;\; is \;\; the curve \;\; y = \frac{1}{x^2} \;\; from \;\; z = 1+i \;\; to \;\; z = 3+\frac{i}{19}$. [6 marks]

(b) Suppose that a function f is analytic in a star D . Suppose further that C is a closed contour lying in D. Prove that $\oint_C f(z)dz = 0$. **[6 marks]**

(c) Determine the value of the contour integral $\oint_{|z|=3} \frac{e^z + \sin z}{z^2 - 25}dz$ where the contour of integration is the circle centre at 0 and with radius 3 followed in the positive (anticlockwise) direction. **[8 marks]**

Question 5 [20 marks]

Given $f(z) = \tan z$

(i) Obtain the Laurent series for the complex function $f(z)$ **[12 marks]**

(ii) Determine the residues of $f(z)$ **[8 marks]**