JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

4TH YEAR SPECIAL RESITS – 2016

MAIN REGULAR

RESIT

COURSE CODE: SMA 202

COURSE TITLE: VECTOR ANALYSIS

EXAM VENUE: LAB 1 **STREAM: (BSc. Actuarial)**

DATE: 06/05/2016 **EXAM SESSION: 2.00 – 4.00 PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY)

a) Given $\vec{r}_1 = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k},\ \vec{r}_2 = 2\underset{\sim}{i} - 4\underset{\sim}{j} - 3\underset{\sim}{k},\ \vec{r}_3 = -\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k},$ find the magnitude of $2\underset{\sim}{r}_1 - 3\underset{\sim}{r}_2 - 5\underset{\sim}{r}_3$. (5 marks)

b) Find a unit vector parallel to the resultant vectors $\underset{\sim}{r}_1 = 2\underset{\sim}{i} + 4\underset{\sim}{j} - 5\underset{\sim}{k},\ \underset{\sim}{r}_2 = \underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$.
. (5 marks)

c) Given the scalar field defined by $\phi(x, y, z) = 3x^2z - xy^3 + 5$, find ϕ at the point $(2, -2, 1)$.
. (4 marks)

d) Find the angle between vectors $\vec{P} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\vec{Q} = 6\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$. (6 marks)

e) If $\vec{S} = 2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$ and $\vec{T} = \underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}$, find $\left|\vec{S} \times \vec{T}\right|$. (5 marks)

f) Given $\vec{R} = \cos t\underset{\sim}{i} + \sin t\underset{\sim}{j} + 2t\underset{\sim}{k}$, find $\left|\dfrac{d^2\vec{R}}{dt^2}\right|$. (5 marks)

QUESTION TWO

a) Determine a unit vector parallel to the plane of $\vec{P} = 2\underset{\sim}{i} - 6\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\vec{Q} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$.
. (6 marks)

b) If $\vec{A} = \underset{\sim}{i} - 2\underset{\sim}{j} - 3\underset{\sim}{k},\ \vec{B} = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ and $\vec{C} = \underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ find $\left|\vec{A} \times \left(\vec{B} \times \vec{C}\right)\right|$. (6 marks)

c) Evaluate $\left(2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right) \times \left(3\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k}\right)$. (4 marks)

d) Find the projection of the vector $2\underset{\sim}{i} - 3\underset{\sim}{j} + 6\underset{\sim}{k}$ on the vector $\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$. (4 marks)

QUESTION THREE

a) If $\vec{Q} = 5t^2\underset{\sim}{i} + t\underset{\sim}{j} - t^3\underset{\sim}{k}$ and $R = \sin t\underset{\sim}{i} - \cos t\underset{\sim}{j}$, find $\dfrac{d}{dt}\left(\vec{Q} \times \vec{R}\right)$. (6 marks)

b) If $\vec{E} = \left(2x^2y - x^4\right)\underset{\sim}{i} + \left(e^{xy} - y\sin x\right)\underset{\sim}{j} + \left(x^2\cos y\right)\underset{\sim}{k}$, find $\dfrac{\partial^2\vec{E}}{\partial y \partial x}$ at the point $(1, -1, 2)$.
. (6 marks)

c) Find the unit tangent vector to any point on the curve $x = a\cos\omega t,\ \ y = a\sin\omega t,$ $z = bt$ where $a,\ b,\ \omega$ are constants. (4 marks)

d) If $\phi(x, y, z) = x^2yz$ and $\vec{F} = xz\underset{\sim}{i} - xy^2\underset{\sim}{j} + yz^2\underset{\sim}{k}$, find $\dfrac{\partial^3}{\partial x \partial y \partial z}\left(\phi\vec{F}\right)$ at the point $(2, -1, 1)$.
. (5 marks)

QUESTION FOUR

a) Find $\nabla\left|\underset{\sim}{r}\right|^3$. (4 marks)

b) If $\vec{F} = \left(3x^2y - z\right)\underset{\sim}{i} + \left(xz^3 + y^4\right)\underset{\sim}{j} - 2x^3z^2\underset{\sim}{k}$, find $\nabla\left(\nabla \cdot \vec{F}\right)$ at the point $(-1, 2, 0)$. (6 marks)

c) If $P = x^2yz,\ \ Q = xy - 3z^2$, find $\nabla \times \left[(\nabla P) \times (\nabla Q)\right]$. (6 marks)

d) Find the unit outward drawn normal to the surface $(x-1)^2 + y^2 + (z+1)^2 = 9$ at the point $(3, -1, 4)$. (4 marks)

QUESTION FIVE

a) If $\vec{P} = (4x^2 + 5y)\underset{\sim}{i} - 12yz\underset{\sim}{j} + 10xz^2\underset{\sim}{k}$, evaluate $\int_c \vec{P} \cdot d\underset{\sim}{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following path c:

i. $x = t,\ y = t^2,\ z = t^3$; (4 marks)

ii. the straight line from $(0,0,0)$ to $(1,0,0)$ then to $(1,1,0)$ and then to $(1,1,1)$;
. (4 marks)

iii. the straight line joining $(0,0,0)$ and $(1,1,1)$. (4 marks)

b) Evaluate $\iint_S \vec{F} \cdot \underset{\sim}{n}ds$, where $\vec{F} = z\underset{\sim}{i} + x\underset{\sim}{j} - 3y^2z\underset{\sim}{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. (8 marks)