**BONDO UNIVERSITY COLLEGE**

**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**

**UNIVERSITY EXAMINATION 2012**

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE) WITH IT

**SPH 205: MATHEMATICAL METHODS FOR PHYSICS II**

## INSTRUCTION

1. Answer question **ONE** in **SECTION A** and any other **TWO** questions from **SECTION B**.
2. Question **ONE** in **SECTION A** carries **30 MARKS** and the questions in **SECTION B** carry **20 MARKS** each.

**Take where necessary**

$$e^z = \sum \frac{z^n}{n!} = 1 + z + \frac{z^2}{2!} + \dots, \qquad |z| < \infty$$

## SECTION A. *Answer ALL questions in this section (30 marks).*

### QUESTION ONE

1.(a) Show that the matrices $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ satisfy the commutation relation $[\mathbf{A}, \mathbf{B}] = \mathbf{C}$. (3 marks)

(b) Solve the differential equation $dy/dx = xy^2$. (3 marks)

(c) Given matrix $A$, find $A^*$, $A^T$ and $A^+$, where

$$A = \begin{pmatrix} 2+3i & 1-i & 5i & -3 \\ 1+i & 6-i & 1+3i & -1-2i \\ 5-6i & 3 & 6 & -4 \end{pmatrix}$$

(3 marks)

(d) Show that $|W\rangle = (4, -1, 8)$ is not a linear combination of $|U\rangle = (1, 2, -1)$ and $|V\rangle = (6, 4, 2)$. (3 marks)

(e) Given $z = x + iy$, find $\cos z$. (3 marks)

(f) Evaluate $(1+i)^4$. (3 marks)

(g) A radioactive isotope decays in such a way that the number of atoms present at a given time $N(t)$, obeys the equation

$$\frac{dN}{dt} = -\lambda N.$$

If there are initially $N_0$ atoms present, find $N(t)$ at later times.

(3 marks)

(h) Find the eigenvalue and eigenvector of the matrix

$$A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$$

(3 marks)

(i) Show that the function $f(z) = 1/_{z-2}$ is analytic everywhere except at $z = 2$ . (3 marks)

(j) Show that the equation $x\, {}^{dy}/_{dx} + (x + y) = 0$ is exact and find its general solution. (3 marks)

## <u>SECTION B.</u> *Answer any TWO questions.*

### <u>QUESTION TWO</u>

2 (a) Show that $(\mathbf{AB})^{+} = \mathbf{B}^{+}\mathbf{A}^{+}$ (6 marks)

(b) Given the Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where $i^2 = -1$. Show that

(i) they are Hermitian (4 marks)

(ii) $\sigma_i^2 = \mathbf{I}$; $i = x, y, z$ . $\mathbf{I}$ is the identity operator. (3 marks)

(iii) $[\sigma_x, \sigma_z] = -2i\sigma_y$ (4 marks)

(c) If $\mathbf{A}$ and $\mathbf{B}$ are square matrix of the same order, then $\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$. Verify this theorem if

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 7 & 2 \\ -3 & 4 \end{pmatrix}$$ (3 marks)

### <u>QUESTION THREE</u>

3. (a) (i) Determine whether the set three vectors $|1\rangle = (2, -1, 0, 3)$, $|2\rangle = (1, 2, 0, 3)$ and $|3\rangle = (7, -1, 5, 3)$ are linearly dependent or independent. (5 marks)

(ii) Determine whether the set S of the four matrices $|1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $|2\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $|3\rangle = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ and $|4\rangle = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ are linearly dependent or independent.

(3 marks)

(b) Expand the vector $|U\rangle = \begin{pmatrix} 1+i \\ \sqrt{3}+i \end{pmatrix}$ in orthonormal basis $|e_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $|e_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. (5 marks)

(c) Let $|U\rangle = (3-4i)|1\rangle + (5-6i)|2\rangle$ and $|W\rangle = (1-i)|1\rangle + (2-3i)|2\rangle$ be set of two vectors expanded in terms of an orthonormal basis $|1\rangle$ and $|2\rangle$.

(i) Evaluate $\langle U|U\rangle$ (2 marks)

(ii) Show that $\langle U|W\rangle = \langle W|U\rangle^*$ (5 marks)

## QUESTION FOUR

4. (a) (i) Derive De Moivre's theorem for roots of complex numbers. (4 marks)

(ii) If $z = e^{i\theta}$, use De Moivre's theorem to show that

$$z^n - \frac{1}{z^n} = 2i\sin(n\theta)$$ (4marks)

(b) (i) Deduce the Cauchy-Riemann condition for the analyticity of a complex function $f(z) = u(x,y) + iv(x,y)$. (7 marks)

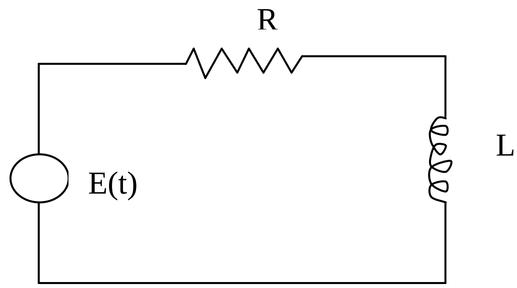
(ii) Given the following functions;

$$e^{3z}(z+1)^{-3}$$

find the Laurent series about the singularity for the functions, name the singularity, and give the region of convergence. (5 marks)

## QUESTION FIVE

5. (a) A typical RC circuit is shown in figure 1.




(i) Find the current $I(t)$ in the circuit as a function of time. (8 marks)

(ii) A solenoid has an inductance of $53mH$ and a resistance of $0.37\Omega$. If the solenoid is connected to a battery, how long will the current take to reach half its final equilibrium value? (3 marks)

(b) (i) State four condition that a function must fulfill in order that it may be expanded as a Fourier series. (4 marks)

(ii) Develop the Fourier series representation of

$$f(t) = \begin{cases} -1 \ \textit{for} \ \tfrac{1}{2}T \le t < 0 \\ -1 \ \textit{for} \ 0 \le t < \tfrac{1}{2}T \end{cases}$$

This is the square wave function which represents the input to an electrical circuit that switches between a high and a low state with time period $T$.

(5 marks)