



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE**

**ACTUARIAL**

**SPECIAL RESIT 2015/2016 ACADEMIC YEAR**

**MAIN REGULAR RESIT**

---

**COURSE CODE: SAC 408**

**COURSE TITLE: RISK MATHEMATICS**

**EXAM VENUE: LAB 1**

**STREAM: (BSc. Actuarial)**

**DATE: 04/05/2016**

**EXAM SESSION: 11.30 – 1.30 PM**

**TIME: 2.00 HOURS**

---

**Instructions:**

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

### **QUESTION ONE [30 marks]**

1) Based on an analysis of past claims, an insurance company believes that individual claims in a particular category for the coming year will have a mean size of \$ 5,000 and a standard deviation of \$ 7,500. Estimate the proportion of claims that will exceed \$ 25,000 assuming total individual claim sizes conform to a log normal distribution. [6 marks]

2) State the aims of for the insurance companies to have policy excess attached to an insurance product. [4 marks]

3. State the 3 characteristics of an investment decision. [3 marks]

4) Briefly describe what you would consider convincing enough for you to lay down an investment (or postpone current consumption) [4 marks]

5a). Using the standard Black Scholes option price formula, calculate the price of a European call on a dividend paying –paying stock with the following features: [5 marks]

Risk free rate: 6% pa (continuously compounded)

Volatility 20% pa

Dividends: 3% (continuously compounded)

Time to expiry: 3 months

Current price of underlying 450 kshs

Strike price 400kshs

b) Use the put call parity relationship to calculate the value of European put for the question above. [3 marks]

c) State the conditions that underlie the standard Black Scholes theory of options pricing. [3 marks]

6). A 3 month European call option with an exercise price of 500 on a share has current price is 480 is currently priced at 42. What would you expect the price to be for a 3 month put option with the same exercise price if the risk free interest rate (continuously compounded) is 6 % and no dividends are payable during the life of the option. (2 marks)

**QUESTION 2 [20 marks]**

1. a) State what would have employers liable under the employers liability insurance. [3 marks]
- b) State the two conditions that must hold for a risk to be insurable. [2 marks]
- c) State 5 other risk factors that would be considered desirable by a general insurance. [5 marks]
- 2a.) Define the terms long-tailed and short-tailed in terms of general insurance. [2 marks]
- b) Describe the 3 categories of financial loss [6 marks]
- 3) Define product liability and give an example [2 marks]

**QUESTION 3 [20 marks]**

- 1) Losses from a certain type of insurance policy are assumed to follow a gamma( $\alpha, \beta$ ) distribution with mean \$ 1,600 and variance ( $\$ 800^2$ )
- a) Determine the values of  $\alpha$  and  $\beta$  [4 marks]
- b) Find the proportion of losses that exceeds \$ 4000 [3 marks]
- 2a.) Let  $X \sim \text{Gamma}(\alpha, \lambda)$ , show that the mgf of X is given by

$$M_x(t) = \left(1 - \frac{t}{\lambda}\right)^{-\alpha} \quad [5 \text{ marks}]$$

- b) Show that if X has a Gamma (10, 4) distribution, then the random variable  $Y=8X$  has a  $\chi^2_{20}$  distribution. Hence approximately the probability that X is greater than 4,375 [3 marks]

4) The aggregate claims arising during each year from a particular type of annual insurance policy are assumed to follow a normal distribution with mean  $0.5P$  and standard deviation  $2.0P$ . Where  $P$  is the annual premium. Claims are assumed to arise independently. Insurers are required to assess their solvency position at the end of each year. A small insurer with an initial surplus of Kshs.200,000 for this type of insurance expects to sell 100 policies at the beginning of the coming year in respect of identical risks for an annual premium of kshs.5,000. The insurer will incur expenses of  $0.1P$  at the time of writing each policy. Calculate the probability that the insurer will prove to be insolvent for this portfolio at the end of the coming year. Ignore

interest [5 marks]

#### **QUESTION 4 [20 marks]**

1). Show that the following utility functions have constant relative risk aversion

co-efficient.

[4 marks]

a)  $u(x) = \ln x$

b)  $u(x) = \alpha x^\alpha$

2) Consider a world in which there are only two risky assets ,A and B, and a risk-free asset F.

The two risky asset are equal in supply in the market; that is  $M = \frac{1}{2}(A + B)$ . It is known that

$r_F = 0.10$ ,  $\sigma_A^2 = 0.04$ ,  $\sigma_{AB} = 0.01$ ,  $\sigma_B^2 = 0.02$  and  $\bar{r}_M = 0.18$  (Assume usual notation)

a) Find the general expression (without substituting values) for  $\sigma_M^2$ ,  $\beta_A$  and  $\beta_B$  [3 marks]

b) According to CAPM, what are the numerical values for  $\bar{r}_A$  and  $\bar{r}_B$  [4 marks]

3) A company is considering two capital investment projects .Project A requires an immediate expenditure of \$1,000,000 and will produce returns of \$ 270,000 at the end of each of the next 8 years .Project B requires an immediate investment of \$ 1,200,000 together with a further expenditure of \$ 20,000 at the end of each of the first three years, will produce returns of \$ 135,000 at the end of each of the sixth, seventh and eighth years

(i) Calculate (to the nearest 0.1%) the IRR per annum for each project. [5 marks]

(ii) Find the NPV of each project on the basis of effective annual interest rate of 10%

[4 marks]

#### **QUESTION 5 [20 marks]**

The Capital Asset Pricing model is assumed to hold in a particular investment market. The total return on a unit invested in asset A in this market has mean 1.15 and standard deviation 0.10. The return on a unit invested risk free is 1.05 and the expected return on a unit invested in the market portfolio is 1.08. You are given that A is an efficient portfolio

a) Derive the equation for the capital market line [6 marks]

b) Calculate the standard deviation of the return on the market portfolio [3 marks]

c) Calculate the  $\beta$  for asset A [5 marks]

d) Asset B has a  $\beta$  of 4 and a standard deviation of return of 0.15. Determine whether B is an efficient portfolio. Give reasons for your answer. [6 marks]