JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

4TH YEAR 1ST SEMESTER 2015/2016 ACADEMIC YEAR

MAIN CAMPUS - RESIT

COURSE CODE: SAS 402

COURSE TITLE: BAYESIAN INFERENCE AND DECISION THEORY

EXAM VENUE: LAB 1 **STREAM: (BSc. Actuarial)**

DATE: 05/05/16 **EXAM SESSION: 11.30 – 1.30PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE COMPULSORY (30MKS)

a) Define the terms Bayesian inference: (2mks)

b) Suppose Y_i is the number of individuals of a particular species observed during a survey i. Assume y_I iid ~Po(λ) where $E(Y_i) = \lambda$. What is the posterior distribution for λ

(5mks)

c) You are interested in the average 1st year G.P.A. of actuarial Science students at JOOUST. You and your three friends all have different views on what the average G.P.A is. You think it will be 47(as first year students don't care about good grades) and your friends think it is 52,57, and 67. Assume that before you see any data, each of these values are equally credible.

(i) What is the parameter of interest? (2mks)

(ii) When doing Bayesian inference, what assumption do you make about this parameter (2mks)

(iii) Given the information, what is the prior distribution of the parameter of interest? (2mks)

d) Taxi cabs are either blue or green. A taxi got an accident and a witness said the car was blue. Witnesses are known to be 80% accurate. If 85% of the taxis on the street were green that day. What is the probability that the taxi cab involved was blue?

(5mks)

e) American Cancer Society estimates that about 1.7% of women have breast cancer. Susan Komen for The Cure Foundation states that mammography correctly identifies about 78& of women who truly have breast cancer. An article published in 2003 suggests that up to 10% of all mammograms are false.

(i) Prior to any testing and any information exchange between the patient and the doctor, what probability should a doctor assign to a female patient having cancer? (2mks)

(ii) If a mammogram yields a positive result, what is the probability that the patient has cancer (5mks)

(iii) Since a positive mammogram doesn't necessarily mean that the patient actually has breast cancer, the doctor might decide to re-test the patient. What is the probability of having breast cancer if this second mammogram also yields a positive result? (5mks)

QUESTION TWO (20MKS)

a) Prove that the bias of a Bayesian estimator $\hat{\mu}$ with normal data $N\left(\bar{y}, \frac{\sigma^2}{n}\right)$ and conjugate prior (m, s^2) is $\left(\frac{\sigma^2}{ns^2 + \sigma^2}\right)(m - u)$

(10mks)

b) Given that claim amounts are uniformly distributed $[0, \theta]$ and the prior density is $\pi(\theta) = \frac{500}{\theta^2}, \theta \triangleright 500$. Two claims $X_1 = 400$ and $X_2 = 600$ are observed. Find the posterior distribution (10mks)

QUESTION THREE (20MKS)

a) A binary communication channel carries data as one of two sets of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as a 1, and a transmitted 1 is sometimes received as a 0. For a given channel, it can be assumed that a transmitted 0 is correctly received with probability 0:95 and a transmitted 1 is correctly received with probability 0.75. Also, 60% of all messages are transmitted as a 0. If a signal is sent, determine the probability that:

(i) a 1 was received; (2mks)

(ii) a 0 was received; (2mks)
(iii) an error occurred; (2mks)
(iv) a 1 was transmitted given than a 1 was received; (2mks)
(v) a 0 was transmitted given that a 0 was received. (2mks)

b) A portfolio of risks is divided into three classes. The characteristics of the annual claim distributions for the three risk classes is as follows:

	Class I	Class II	Class III
Annual Claim Number Distribution	Poisson mean 1	Poisson mean 2	Poisson mean 5

50% of the risks are in Class I, 30% are in Class II, and 20% are in Class III. A risk is chosen at random from the portfolio and is observed to have 2 claims in the year. Find the probability that the risk was chosen from Class I.

QUESTION FOUR (20MKS)

a) Annual number of claims for a policy holder has binomial distribution

$$P(X/q)=\binom{2}{x}q^x(1-q)^{2-x}, x=0,1,2$$

Prior distribution is given by $\pi(q)=4q^3, 0<q<1$

The policy holder had one claim in each of years1 and 2. Determine the Bayesian estimate of the number of claims for year 3. (10mks)

b) You are given:

(i) Losses on a company's insurance policies follow a pareto distribution with probability density function:

$$f(x|\theta)=\frac{\theta}{(x+\theta)^2}, \quad 0<x<\infty$$

(ii) For half of the company's policies $\theta=1$, while for the other half $\theta=3$. For a randomly selected policy, losses in year 1 were 5. Determine the posterior probability that losses for this policy in year 2 will exceed 8 (10mks)

QUESTION FIVE (20MKS)

a) Suppose that an insured population consists of 1500 youthful drivers and 8500 adult drivers. Based on experience, suppose that we have derived the following probabilities that an individual driver will have n claims in a one year period.

n	Youth	Adult
0	0.50	0.80
1	0.30	0.15
2	0.15	0.05
3	0.05	0.00

It is found that a particular randomly chosen policy has exactly one claim on it in the past year.
Find the probability that the policy is for a driver who is a youthful driver. (10mks)

b) You are given:

(i) The probability that an insured will have at least one loss during any year is p.

(ii) The prior distribution for p is uniform on $[0, 0.5]$.

(iii) An insured is observed for 8 years and has at least one loss every year.

Determine the posterior probability that the insured will have at least one loss during Year 9.

(10mks)