JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THEDEGREE OF BACHELOR OF EDUCATION (SCIENCE)

4^{TH} YEAR 2ND SEMESTER

MAIN

REGULAR

COURSE CODE: SPH 402

COURSE TITLE: STATISTICAL MECHANICS

EXAM VENUE: **STREAM: (BED SCI)**

DATE: **EXAM SESSION:**

TIME: 2:00HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful constants

$h = 6.63\times10^{-34}\,Js$

$c = 3\times10^{8}\,m/s$

$k_B = 1.38\times10^{-23}\,JK^{-1}$

QUESTION 1 (30 MARKS)

(a) Define the following terms.

(i) Statistical Physics **(1 mark)**

(ii) Canonical ensemble **(1 mark)**

(iii) Grand canonical ensemble **(1 mark)**

(iv) Phase space **(1 mark)**

(b) The Hamiltonian of a given system in phase space is given by $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$

Determine the Hamiltonian canonical equations of motion. **(3 marks)**

(c) Two bosons are in an energy level with degeneracy of 3. Determine the number of microstates of these bosons. **(3 marks)**

(d) State **TWO** features of particles obeying Maxwell-Boltzmann statistics.

(2 marks)

(e) By using Helmholtz free energy, derive expressions for entropy, pressure and chemical potential. **(4 marks)**

(f) Derive the equation for the partition function of a spin-$\frac{1}{2}$ paramagnetic system.

(3 marks)

(g) Explain the formation of Bose-Einstein condensate. **(2 marks)**

(h) Show that the average internal energy in a canonical ensemble is given by

$\overline{E} = \frac{1}{Z}\sum_i \varepsilon_i e^{-\beta\varepsilon_i}$ where each symbol has its usual meaning. **(3 marks)**

(i) Explain the term black body as used in Statistical Physics **(2 marks)**

(j) Determine the percentage error in the use of Stirling's formula to calculate $\ln N!$ for a system of $N = 10000$ particles. **(4 marks)**

Attempt any TWO questions in this section

QUESTION 2 (20 MARKS)

(a) Show that the Maxwell-Boltzmann distribution function is given by

$$\frac{n_i}{g_i} = \frac{N}{Z} e^{-\frac{\varepsilon_i}{k_B T}}$$ **(12 marks)**

(b) Blackbody radiation in a box of volume V and at temperature T has internal energy $U = \sigma V T^4$ and pressure $P = \frac{1}{3}\sigma T^4$, where σ is the Stefan-Boltzmann constant. Determine the entropy and chemical potential in terms of U.

(8 marks)

QUESTION 3 (20 MARKS)

(a) Consider a system of 6 particles that obey Fermi-Dirac statistics with a a degeneracy of 3 in each energy level. Given that the energy levels are equally spaced and the total energy per macrostate is $E = 6\varepsilon$ where $E_1 = \varepsilon$, $E_2 = 2\varepsilon$ and so on, determine the statistical weight of the system. **(10 marks)**

(b) Obtain an expression for the entropy of a thermodynamic system in terms of the statistical weight. **(10 marks)**

QUESTION 4 (20 MARKS)

(a) Show that the energy density in blackbody radiation is given by

$$u(\lambda,T)=\frac{8\pi hc}{\lambda^5\left(e^{\frac{hc}{kT\lambda}}-1\right)}d\lambda$$ **(15 marks)**

(b) Show that the maximum value of the wavelength for which $u(\lambda,T)$ obtained in 4 (a) is maximum is obtained from $\lambda_m T = 2.9mmK$ **(5 marks)**

QUESTION 5 (20 MARKS)

(a) Derive the average internal energy of a system of N-independent particles in the form $\overline{U} = KT^2\left(\frac{\partial \ln Z}{\partial T}\right)$ **(7 marks)**

(b) Determine the conditions under which a composite isolated system C resulting interaction of two isolated systems A and B attains thermodynamic equilibrium.

(13 marks)