JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION

3RD YEAR 1ST SEMESTER 2018/2019

MAIN REGULAR

COURSE CODE: SPH 313

COURSE TITLE: MECHANICS

EXAM VENUE: **STREAM: (B.Ed Sc)**

DATE: **EXAM SESSION: 8.00 A.M. – 10.00 A.M.**

TIME: 2:00 HRS

INSTRUCTIONS:

1. **Attempt question 1 (compulsory) and ANY other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful constants

gravitational acceleration, $g, = 9.8$ m s^{-2}
velocity of light in free space $= 3.0 \times 10^8$ m/s
mass of an electron $= 9.11 \times 10^{-31}$

Question 1

(a) (i) Explain what you understand by a conservative force field, giving a necessary and sufficient condition for a force field to be conservative.

(2 Marks)

(ii) A single conservative force $\vec{F} = (6.0x - 12)\hat{\imath}$ N, where x is in meters acts on a particle moving along the x-axis. Determine an expression for the kinetic energy as a function of x, if the particle was initially at the origin.

(4 Marks)

(c) Consider a system of N interacting particles with masses m_i, position vectors $\vec{r}_i$, and momenta $\vec{p}_i = m_i\dot{\vec{r}}_i$ $(i = 1, 2, \cdots, N)$ relative to an inertial frame of reference. The particles are also subject to external forces $\vec{F}_i^{(e)}$, and the masses m_i are assumed to be constant. Show that the equation of motion of the center of mass vector is

$$M\ddot{\vec{R}} = \vec{F}^{(e)}$$

(4 Marks)

(d) Three forces act on a mass that moves at a constant velocity. Two of the forces are $\vec{F}_1 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ N and $\vec{F}_2 = -5\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ N. Find the third force.

(3 Marks)

(d) An observer on Earth sees a spaceship at an altitude of 4360 km moving towards the Earth with a speed of $0.980c$, where c is the speed of light in free space. Find the distance from the space ship to the Earth as measured by the captain of the spaceship.

(3 Marks)

(e) A block of metal of mass m_1 lies on a horizontal table. It is attached to a mass m_2 by a light string passing over a frictionless pulley at the edge of the table (see Figure 1). The

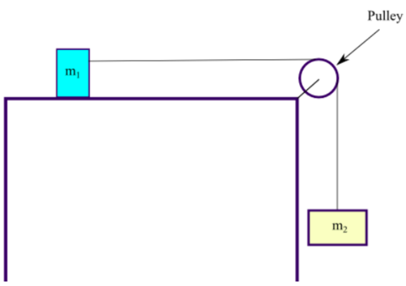

Figure 1:

coefficient of sliding friction between the block and the table is μ.

(i) Determine the acceleration of the system, a, in terms of μ, m_1, m_2 and the gravitational acceleration, g.

(ii) Hence obtain an expression for the tension in the string.

(iii) Given that $m_1 = 1.5$ kg, $m_2 = 0.4$ kg and $\mu = 0.1$, find the acceleration of the system.

(5,2,2 Marks)

(f) When a system of particles is acted upon by a number of forces, it changes from configuration 1 to configuration 2. Derive an expression for the work done by all the forces.

(3 Marks)

(g) Write down Hamilton's equations for the Hamiltonian

$$H(t,p,q) = \frac{p^2}{2m} - mCtq$$

where C is a constant.

(2 Marks)

Question 2

(a) Explain the advantage of using Lagrangian mechanics compared to Newtonian mechanics in solving mechanics problems.

(2 Marks)

(b) Determine the equations of motion of the masses of Atwood machine by the Lagrangian method.

(8 Marks)

(c) A system is described by the single generalized coordinate q and the Lagrangian $L(q,\dot{q})$.

(i) Define the generalized momentum associated with q and the corresponding Hamiltonian, $H(q,p)$. What characteristic feature should the Lagrangian function have for a generalized momentum to be a constant of motion?

(ii) Derive Hamilton's equations from Lagrange's equations of the system. Explain the significance of the Hamiltonian when used to describe a system.

(10 Marks)

Question 3

(a) A spaceship of rest length 130 m races past a timing station at a speed of $0.740c$.

(i) What is the length of the spaceship as measured by the timing station?

(ii) What time interval will the station clock record between the passage of the front and back ends of the ship?

(6 Marks)

(iii) Give an account of the "twin paradox", and explain how it is resolved.

(4 Marks)

(b) Muons have a mean lifetime of 2.2×10^{-6} s when at rest. They are produced at an altitude of 10 km and travel at $0.995c$ toward the earth. Find

(i) The mean lifetime measured on earth.

(ii) The time taken to reach ground level in the earth frame.

(iii) The time taken to reach ground level in the particle's frame.

(10 Marks)

Question 4

(a) (i) Determine the energy required to give an electron a speed of $0.90c$, starting from rest.

(3 Marks)

(ii) Investigate whether the force field defined by

$$\vec{F} = (2y + 8xy^3)\hat{\mathbf{i}} + (2x + 12x^2y^2)\hat{\mathbf{j}}$$

is conservative.

(6 Marks)

(b) A particle of mass 3 kg moves in a force field dependent on time t given by

$$\vec{F} = 24t^2\hat{\mathbf{i}} + (36t - 21)\hat{\mathbf{j}} - 15t\hat{\mathbf{k}}$$

If at $t = 0$ the particle is located at $\vec{r}_0 = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and has velocity $v_0 = 6\hat{\mathbf{i}} + 15\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$, find the position at any time t.

(8 Marks)

. **(c)** From the definition of linear momentum, derive the mathematical expression of Newton's second law.

(3 Marks)

Question 5

(a) Two masses are connected by a massless rod of length l (see Figure 2). The mass m_1 moves without friction along a horizontal line, while the mass m_2 moves in a plane passing through the line.

(i) Find the Lagrangian of the system.

(ii) Hence find the equations of motion for small oscillations about equilibrium.

(6, 6 Marks)

(b) Derive the formula for time dilation using the inverse Lorentz transformation.

(6 Marks)

(c) The period of a pendulum is measured to be 4.00 s in the inertial frame of the pendulum at the surface of the earth. Find the period as measured by an observer moving at a speed of $0.96c$ with respect to the pendulum.

(2 Marks)

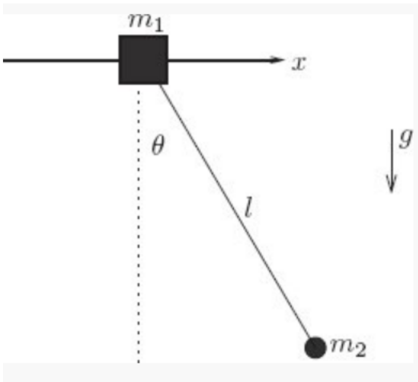

Figure 2: