# JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES <br> UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE) <br> $4^{\text {TH }}$ YEAR $1^{\text {ST }}$ SEMESTER 2018/2019 ACADEMIC YEAR <br> MAIN <br> REGULAR 

COURSE CODE: SPH 410
COURSE TITLE: ELECTRODYNAMICS
EXAM VENUE:
STREAM: EDUCATION

DATE:
EXAM SESSION:
TIME: 2:00 HRS

## Instructions:

1. Answer question 1 (Compulsory) and ANY other 2 questions.
2. Candidates are advised not to write on the question paper.
3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

Useful constants

## SECTION A

## QUESTION 1 (30 MARKS)

a) Let $F_{1}=x^{2} \hat{z}$ and $F_{2}=x \hat{x}+y \hat{y}+z \hat{z}$ calculate the divergence and curl of $F_{1}$ and $F_{2}$. Which one can be written as the gradient of a scalar? Find a scalar potential that does the job which can be written as the curl of a vector? Find a suitable vector potential.
(6 marks)
b) Suppose $v=\left(2 x z+3 y^{2}\right) \hat{\boldsymbol{y}}+\left(4 y z^{2}\right) \hat{\boldsymbol{z}}$. Check Stokes' theorem for the square surface shown in the Fig 1.1 below:

## marks)



Figure 1.1:
c) A non-conducting rod of length $l$ with a uniform charge density $\lambda$ and a total charge $Q$ is lying along the $x$-axis, as illustrated in Figure 1.2. Compute the electric field at a point $P$, located at a distance $y$ from the center of the rod along its perpendicular bisector.
(5 marks)


Figure 1.2:
d) i) State the Biot-Savart's law
(2 marks)
ii) Find the magnetic field a distance $s$ from a long straight wire carrying a steady current $I$
(3 marks)
e) An electromagnetic plane wave has an electric field given by

$$
\overrightarrow{\boldsymbol{E}}=(250 \mathrm{~V} / \mathrm{m}) \cos \left(\frac{2 \pi}{3} x-2 \pi \times 10^{4} t\right) \hat{\mathbf{k}}
$$

Where $x$ and $t$ are in SI units and $\hat{\mathbf{k}}$, is the unit vector in the $+z$-direction. The wave is propagating through ferrite, a ferromagnetic insulator, which has a relative magnetic permeability $k_{m}=1000$ and dielectric constant $k_{e}=10$.
i) What direction does this wave travel?
ii) What is the wavelength of the wave (in metres)?
iii) What is the frequency of the wave (in Hz )?
iv) What is the speed of the wave (in $\mathrm{m} / \mathrm{s}$ )?

## SECTION B

## QUESTION 2 (20 MARKS)

a) Find the magnetic field a distance $z$ above the center of a circular loop of radius R , which carries a steady current $I$.
(6 marks)
b) Show that the reflection coefficient for radiation at normal incidence
(6 marks) from free space on to a plane surface of material of refractive index $\left(n_{R}-i n_{x}\right)$ is

$$
R=\frac{\left(n_{R}-1\right)^{2}+n_{I}^{2}}{\left(n_{R}+1\right)^{2}+n_{I}^{2}}
$$

c) A laser beam, having a power of 100 W and a diameter of 1 mm , passes through a glass window of refractive index 1.59. Find the peak values of the electric and magnetic fields of the laser beam in the:
i) air, and
(4 marks)
ii) glass.

## QUESTION 3 (20 MARKS)

a) Use a rectangular Cartesian coordinates to show that in a free space where there is no charge the following equation applies: $\operatorname{div} E=0$
b) Electrostatic field is conservative; explain this statement; hence show that

$$
\begin{equation*}
\text { curl } \operatorname{grad} \psi=0 \tag{5marks}
\end{equation*}
$$

c) A dipole consists of two equal but opposite charges, $+q$ and $-q$, separated by a distance $2 a$, as shown in Fig. 3.1


Fig. 3.1: Electric dipole
i) define the dipole moment vector
(1 mark)
ii) For the point $P$ lying in the $y-x$ plane, give the first order of potential of the dipole.
iii) Determine the component of the electric field $E_{y}$ and $E_{x}$ in the $x-y$ plane
(5 marks)
iv) What is the potential of the dipole at a point lying along $x$-axis?
(0.5 mark)
v) On which axis is the maximum potential; calculate the maximum potential if $a=5.0 \mathrm{~cm}$ and $Q=15 \mu \mathrm{C}$
(0.5 mark)

## QUESTION 4 (20 MARKS)

a) Derive Maxwell's equation from Coulomb's, Amperes' and Faradays' laws. Modify the equation to satisfy the conditions of slow varying fields with time.
b) A linearly polarized monochromatic EM plane waves in free space (without charges) has electric field given by the following expression $E_{y}=E_{o} \cos (\omega t-k x)$
i) Determine the corresponding magnetic field component of the wave
(2 marks)
ii) Show that the electric and magnetic field components bear a constant ratio; give the ratio.
iii) The amplitude of the electric field in the plane wave is $0.2 \mathrm{Vm}^{-1}$. Calculate the amplitude of the magnetic field component. (2 marks)
iv) Prove that there is no electric and magnetic field components in the direction of propagation of the wave.
(2 marks)
c) What is skin effect? Calculate the skin depth for a transmission line made of copper material, with $\mu=1$ and conductivity $\sigma=5.9 \times 10^{7}$ at a frequency of $f=50 \mathrm{MHz}$.
(4 marks)
d) The resistance to direct current of a copper wire of diameter $\quad d=5.0 \mathrm{~mm}$ is $8 \times 10^{-4} \Omega \mathrm{~m}^{-1}$. Calculate the resistance of the wire at the frequency of, $f=50 \mathrm{MHz}$.
(2 marks)

## QUESTION 5 (20 MARKS)

a) Derive Poynting vector for a plane and monochromatic wave in a free space without medium; and give its physical meaning.
b) Modify your equation for the Poynting vector to satisfy the conditions for the presence of an isotopic medium with a relative permittivity $(\varepsilon)$ and relative permeability $(\mu)$
(4 marks)
c) Calculate, both magnetic and electric fields amplitude at a point 10 km a way from a $1 \mathrm{~kW} \quad$ radio transmitter.
d) If the amplitude of electric field in a plane wave is $0.1 \mathrm{Vm}^{-1}$, a few kilometers away from a radio transmitter, calculate the magnitude of the Poynting vector at the place. If the radio waves were transmitted through water with $\varepsilon=80$ and $\mu=1$, what would be the difference in magnitude of the Poynting vector.

