JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

1^{ST} YEAR 1^{ST} SEMESTER 2018/2019

MAIN REGULAR

COURSE CODE: SPH 802

COURSE TITLE: CLASSICAL ELECTRODYNAMICS

EXAM VENUE: **STREAM: (M.Sc)**

DATE: **EXAM SESSION:**

TIME: 3:00 HRS

INSTRUCTIONS:

1. **Attempt question 1 (compulsory) and ANY other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

You may take:

gravitational acceleration, $g, = 9.8$ m s^{-2}

(i) $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$

(ii) $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

(iii) $\nabla \times \nabla u = 0$

(iv) $\nabla \cdot (\nabla \times \vec{A}) = 0$

(v) Divergence theorem
$\oint_s \vec{A} \cdot \hat{n}\, \mathrm{d}a = \int_v \nabla \cdot \vec{A}\, \mathrm{d}\tau$

(vi) Stoke's theorem
$\oint_c \vec{A} \cdot \hat{n}\, \mathrm{d}l = \int_s \mathrm{curl}\, \vec{A} \cdot \hat{n}\, \mathrm{d}a$

(vii) $\nabla \left(\frac{1}{R}\right) = -\nabla' \left(\frac{1}{R}\right) = -\frac{\hat{R}}{R^2} = -\frac{\vec{R}}{R^3}$

(viii) Gradient theorem
$\int_a^b (\nabla f) \cdot \mathrm{d}l = f(b) - f(a)$

Question 1

(a) Using the vector field

$$\vec{F}(x, y, z) = 5xyz\hat{\mathbf{i}} + y^2\hat{\mathbf{j}} + yz\hat{\mathbf{k}}$$

verify Stoke's theorem by integrating over the open surface S defined by the five sides of a cube measuring 1 m on a side and about the closed line l bounding S.

(4 Marks)

(b) Find the electric field a distance z above the midpoint of a straight line segment of length $2L$ which carries a uniform line charge λ.

(4 Marks)

(c) A metal sphere of radius a carries the charge Q. It is surrounded out to radius b by linear dielectric material of permittivity ϵ. Find the potential at the center relative to infinity.

(4 Marks)

(d) **(i)** A radially dependent volume charge density $\rho_v = 50r^2$ C/m^3 exists within a sphere of radius $r = 0.05$ m. Find the total charge q contained within the sphere.

(ii) The same sphere in (d)(i) is now covered with angularly dependent surface charge density $\sigma = 2 \times 10^{-2} \cos^2 \theta$ C/m^2. Find the total charge on the sphere.

(6 Marks)

(e) **(i)** Consider a plane wave travelling in the positive z-direction with the electric field at all times in the y-direction. Draw a schematic diagram to illustrate linear, elliptical and circular polarization of a plane wave in three dimensions.

(ii) Draw a flow chart showing the steps to develop a method for the transmission of transverse electric waves in a hollow rectangular waveguide.

(4 Marks)

(f) Use the principle of conservation of charge to derive the equation of continuity.

(4 Marks)

(g) Show that the electric field $\vec{E}$ in terms of the vector potential $\vec{A}$ and scalar potential V can be written as

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

(4 Marks)

Question 2

(a) (i) Explain what is meant by polarization.
(ii) State the possible causes of polarization in dielectric materials.

(4 Marks)

(b) (i) Show that a volume charge density ρ_b and a surface charge density σ_b arising from bound charges of a dielectric are given by $\rho_b = -\nabla \cdot \vec{P}$ and $\sigma_b = \vec{P} \cdot \hat{n}$.

(3 Marks)

(ii) Show that the total bound charge of a polarized dielectric of finite extent is always zero.

(3 Marks)

(c) (i) Find the integral relation for the dielectric displacement vector in a dielectric.
(ii) Classify dielectrics on the basis of functional relationship between the polarization and dielectric fields.

(6 Marks)

Question 3

(a) Use Ampere's law in integral form to derive Maxwell's equation

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

(4 Marks)

(b) (i) Show that $\nabla \cdot \vec{B} = 0$.
(ii) Find $A(\vec{r})$ for an ininitely long filamentary wire which carries a current I up the z-axis.

(3,5 Marks)

(c) (i) Using the Poynting vector $\vec{S} = \vec{E} \times \vec{H}$, show that the total power flux over a closed surface is given by

$$-\oint_s \vec{S} \cdot \mathrm{d}\vec{s} = \frac{\partial}{\partial t} \int_v \left[\frac{\vec{H} \cdot \vec{B}}{2} + \frac{\vec{E} \cdot \vec{D}}{2} \right] \mathrm{d}\tau + \oint_v \vec{J} \cdot \vec{E} \, \mathrm{d}\tau$$

where the symbols have their usual meanings.

(6 Marks)

(ii) Using Poynting theorem, evaluate the total power flux entering the closed surface s embracing a length l of a wire of radius a carrying a direct current I.

(4 Marks)

Question 4

(a) A spherical shell of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω. Find the vector potential it produces at a radial distance $\vec{r}$.

(12 Marks)

(b) A magnetic field $\vec{B}$ is applied to a cube of magnetic material b m on a side such that magnetization density $\vec{M}$ is z-directed and varies linearly along the x-axis according to $\vec{M} = 10x\hat{e}_z$ A/m.

(6 Marks)

(c) Distinguish between linear and non-linear dielectrics.

(2 Marks)

Question 5

(a) A system of charged particles within a finite volume V is moving with a velocity $\vec{v}$, where both the electric and magnetic fields $\vec{E}$ and $\vec{B}$ exist. Show that the linear momentum is conserved and can be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{P}_{\text{mech}} + \vec{P}_{\text{field}}\right) = \oint_s \sum_\beta T_{\alpha\beta}\hat{n}_\beta \mathrm{d}a$$

where $T_{\alpha\beta}$ is the Maxwell stress tensor and $\hat{n}_\beta$ is the outward normal unit vector.

(10 Marks)

(b) A sphere of radius R, centered at the origin, carries charge density

$$\rho(r,\theta) = k\frac{R}{r^2}(R - 2r)\sin\theta$$

where k is a constant and r and θ are spherical coordinates. Find the approximate potential for points on the z-axis, far from the sphere.

(10 Marks)