



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE**

**ACTUARIAL**

**2<sup>ND</sup> YEAR 1<sup>ST</sup> SEMESTER 2018/2019 ACADEMIC YEAR**

**REGULAR (MAIN)**

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**COURSE CODE: SAC 201**

**COURSE TITLE: FINANCIAL MATHEMATICS I**

**EXAM VENUE:**

**STREAM: (BSc. ACTUARIAL SCIENCE)**

**DATE:**

**EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION 1 [COMPULSORY] [30 Marks]**

(a) Calculate the nominal rate of discount per annum which is equivalent to;

(i) an effective interest of 1% per quarter. **[3 Marks]**

(ii) a force of interest of 5% per annum. **[3 Marks]**

(iii) a nominal rate of discount of 4% per annum. **[3 Marks]**

(b) Define accumulation factor  $A(t, t + h)$  and the nominal rate of interest  $i_h(t)$  and hence or otherwise derive the formula for the force of interest per unit time  $\delta(t)$ . **[5 Marks]**

(c) A continuous payment stream is such that the level rate of payment in year  $t$  is  $100 \times 1.05^{t-1}$ . Calculate the present value of the payment stream as at its commencement date assuming a rate of interest of 10% per annum.

**[4 Marks]**

(d) You are offered a loan of Kshs 10,000 with no payments for 6 months, the Kshs.600 per month for 1 year, and Kshs.500 per month for the following year. What annual effective rate of interest does this loan charge?

**[4 Marks]**

(e) The force of interest  $\delta(t)$  is a function of time, and at any time  $t$ , measured in years, is given by

$$\delta(t) = 0.005t + 0.0001t^2$$

for all  $t$ .

At  $t = 8$ , calculate the accumulated value of an investment of Kshs.100,000

made at time  $t = 0$  [4 Marks]

(e) Suppose that  $l$  and  $m$  are positive integers. Express  $i^{(m)}$  in terms of  $l$ ,  $m$  and  $d^{(l)}$ . Hence find  $i^{12}$  when  $d^{(4)} = 0.057847$ . [4 Marks]

## QUESTION 2 [20 Marks]

(a) An investor pays Kshs.8,000 at the start of each month into a 25-year savings plan. The contributions accumulate at an effective rate of interest of 3% per half-year for the first 10 years, and at a force of interest of 6% per annum for the final 15 years.

Calculate the accumulated amount in the savings plan at the end of 25 years. [8 Marks]

(b) For a rate of interest of 7% per annum, convertible monthly, calculate:

(i) the equivalent rate of interest per annum convertible half yearly.

[6 Marks]

(ii) the equivalent rate of discount per annum convertible monthly. [6 Marks]

**QUESTION 3 [20 Marks]**

The force of interest  $\delta(t)$  is a function of time, and at any time  $t$ , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.24 - 0.02t & 0 < t \leq 6 \\ 0.12 & 6 < t \end{cases}$$

(a) Derive, and simplify as far as possible, expressions in terms of  $t$  for the present value of a unit investment made at any time,  $t$ . You should derive separate expressions for each time interval  $0 < t \leq 6$  and  $6 < t$ . **[8 Marks]**

(b) Determine the discounted value at time  $t = 4$  of an investment of Kshs.100,000 due at time  $t = 10$ . **[3 Marks]**

(c) Calculate the constant nominal annual interest rate convertible monthly implied by the transaction in part (b). **[3 Marks]**

(d) Calculate the present value of a continuous payment stream invested from time  $t = 6$  to  $t = 10$  at a rate of  $\rho(t) = 20e^{0.36+0.32t}$  per annum.

**[6 Marks]**

**QUESTION 4 [20 Marks]**

A loan of Kshs.80,000 is repayable over 25 years by level monthly installments in arrears of capital and interest. The repayments are calculated using an effective rate of interest of 8% per annum. Calculate:

(a) The capital repaid in the first monthly installment. [5 Marks]

(b) The total amount of interest paid during the last six years of the loan.

[5 Marks]

(c) The interest included in the final monthly payment. [5 Marks]

(d) Explain how your answer to (b) would alter if, under the original terms of the loan, repayments had been made less frequently than monthly.

[5 Marks]

**QUESTION 5[20 Marks]**

An investment project gives rise to the following cash flows. At the beginning of each of the first three years Kshs.180,000 will be invested in the project. From the beginning of the first year until the end of the twenty-fifth year, net revenue will be received continuously. The initial rate of payment of net revenue will begin at Kshs.25,000 per annum. The rate of payment is assumed to grow continuously at a rate of 6% per annum

effective.

(a) Calculate the net present value of the project at an effective rate of interest of 7% per annum. **[7 Marks]**

(b) Calculate the discounted payback period of the project at an effective rate of interest of 7% per annum. **[6 Marks]**

(c) Calculate the annual effective rate of growth of net revenue which would be required if the project is to have a zero net present value at an effective rate of interest of 7% per annum. **[6 Marks]**